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PUBLIC POLICY AND ENDOGENOUS GROWTH

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ABSTRACT

The simple model presented shows that the behavior of debt and consumption are related to the rates of growth of the labour force, the discount rate and world interest rate. We show that an increase in the shadow price of debt causes the composite price to fall then eventually increase to bring the the capital stock to a lower magnitude. The impact of debt on the current account depends on whether the analysis is in the transition or or steady state.

This study was originally undertaken as a study in dynamic optimization with Prof. David Zervos.

1 Introduction

This paper extends the literature by examining the behavior of endogenous growth when a 'small' economy faces stochastic terms of trade and infrastructure is financed through external debt rather than taxes. The research agenda on endogenous growth has neglected any analysis of the role of external debt . Barro (1992), Sala-i-Martin (1990), Barro (1988) , in exploring the constant returns to scale branch of the literature uses a fairly broad definition of the capital stock to present an array of models with tax financed public expenditure .

The hypothesis that differences in national public policies are the explanatory factors in the divergence in long run growth rates between different countries is discussed in King and Rebelo (1990) .The authors utilise a two-sector endogenous growth model to show that large differences in long run growth rates that can arise due to the incentive effect of taxation . The fiscal policy side of the literature has a richer history than that of endogenous growth , going back to Fisher (1920) and Lerner,(1946).

The modern branch of this literature is typified by Anderson and Young (1992). In this paper, unanticipated fluctuations in current consumption, productivity or external interest rates are used to characterise the optimal time profile of a tax on consumption. Lapan and Walters (1990) develop an overlapping generations model in which fertility is optimally determined . Government debt represents a tax on future generations and it is shown that the first best policy is to eliminate debt as it reduces welfare by distorting fertility decisions.

The terms-of-trade literature is just as rich , starting with Laursen and Metzler (1950) and Harberger (1950).The former used simple expenditure/income models to show the effects of changes in the terms-of-trade on the current account. The latter exploited the Keynesian consumption function to show how the terms-of-trade can deteriorate the current account by reducing real income and savings. Obstfeld(1982) challenged whether the behavior exhibited in the both of these papers were consistent with intertemporal utility maximisation . By using Uzawa endogenous preferences Obstfeld showed that a permanent deterioration in the terms-of-trade could improve the current account .

The current research program in this area involves coupling of the infinite horizons model of Blanchard(1985) with the Q-theory of investment to show the dynamic interrelation of investment ,saving and growth to term-of-trade.

2 The Model

An economy operating under small country condition produces an output Q using domestic inputs capital K, labour L, and an input Ω,which it imports and uses in fixed proportion δ. The production function for this economy can be written as

$$Q = \min(f(K, L), \Omega/\delta). \tag{1}$$

To obtain the net output of this economy we note the following if, σ is the share of imports in output then

$$Q(1 - \delta\sigma) = f(K, L) \tag{2}$$

With the inclusion of a government sector, net output can be interpreted as

$$(1 - \delta\sigma)(f(K, L) = (1 - \delta\sigma)C + I^* + G + (X - \sigma\Omega). \tag{3}$$

Following Takayama(1985)

$$I^* = I - \rho I/K.$$

The capital accumulation process in this economy is governed by

$$I = \eta K. \tag{4}$$

where η is the rate of depreciation. In per capita terms[1] , the relationship between investment

[1] Lower case letters are used to represent per capita values.

I, the capital stock and depreciation can be written as

$$k = i - (\eta + n)k. \quad (5)$$

The government in this economy faces the following budget constraint

$$G = \gamma[M/P] + r\Pi. \quad (6)$$

M/P is the money supply, r is the world rate of interest and Π is the country's international reserves.

The money market equilibrium is found by setting $M_d = M/P$. where M_d is real balances. The deamand for real balances takes the form

$$M_d = \gamma(\hat{r})\hat{W}^2. \quad (7)$$

$\hat{W}$ is real wealth, it consists of real balances and discounted output i.e. $\hat{W} = \gamma(\hat{r})(M/P + Q/\delta)$. This implies

$$M/P = \frac{\gamma(\hat{r})}{1-\gamma(\hat{r})} = \Lambda\gamma(\hat{r})Y/\delta. \quad (8)$$

The government budget constraint can now be viewed as

$$G = \Lambda\gamma(\hat{r})Y/\delta + r\Pi. \quad (9)$$

On the debt side, we represent the growth of external debt as

$$\tilde{D} = D + Rd \quad (10)$$

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[2] $\gamma'(\hat{r}) < 0, \gamma''(\hat{r}) > 0$.

The stock of debt is D and Rd is the accrued interest payments.

The balance of payment similarly can be viewed as

$$r\Pi + X = \sigma\Omega + Rd \tag{11}$$

Equation (10) can be written in per capita terms and substituted along with (9) and (11) into (3) to express net output as

$$(1-\delta\sigma)Y = c + (i - \rho i/k) + \Lambda\gamma(\hat{r})Y/\delta + r\Pi + \sigma\Omega + Rd.(12) \tag{12}$$

Rearranging (12) gives

$$(1-\delta\sigma)f(k)(1-\Lambda\gamma(\hat{r})Y/\delta) = c + i(1-\rho i/k) + rd - \sigma\Omega \tag{13}$$

2.1 The Maximisation Problem

All of our n identical individuals face the problem of maximising utility with respect to debt and subject to equations of motion of capital and debt , this can be written as

$$\max \int_0^\infty U(c)e^{-\delta t}dt \tag{14}$$

and the intertemporal constraint is

$$f(k) = \frac{c = i(1+\rho/k) + rd - a}{(1-\delta\sigma)(1-\Lambda\gamma(\hat{r})Y/\delta)} \tag{15}$$

We write the current value Hamiltonian for our problem as

$$H() = U(c) + v(i - (\eta + n)k) + \chi(a - rd). \tag{16}$$

where the multipliers on the constraints of capital accuumulation ise v and that for debt isχ.

The first order conditions are:

$$\frac{\partial U}{\partial c}\frac{\partial c}{\partial i} + v = 0 \tag{17}$$

$$v + \hat{U}(c) + (1 + \rho(i/k) + i/k\rho k(i/k) = 0 \tag{18}$$

$$i\dot{v} = -\frac{\partial H}{\partial v} + \delta v \tag{19}$$

$$\chi = U'(c) \tag{20}$$

$$\dot{\chi} = -\frac{\partial H}{\partial k} \tag{21}$$

$$\lim_{t\to\infty} e^{-\delta t}\chi dt = 0 \tag{22}$$

$$\lim_{t\to\infty} e^{-\delta t}\upsilon k = 0 \tag{23}$$

The transversality conditions in (22) rule out the possibility of Ponzi schemes and resultantly the external debt for our economy is paid off. For similar reasons (23) the capital stock will not grow without bounds.

If we view the shadow price of debt as the utility denominated value of debt, then we can solve the initial condition to show

$$\chi = (\delta + n)U'(c) + rU'(c). \tag{24}$$

By gathering terms and integrating

$$\int_0^\infty \chi = \chi_0 e^{(\sigma+n-r)}. \tag{25}$$

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Equation (24) gives us the means to solve for the consumption path. (20) expresses the marginal utility of consumption as being equal to the shadow price of debt . This implies

$$\frac{d\chi}{dt} = -U"(c) = (\sigma + n - r)\chi \tag{26}$$

By integrating and substituting, we conclude that

$$\frac{dc}{dt} = \frac{(\sigma + n - r)}{\frac{d}{dc}[\log U'(c)]} \tag{27}$$

The optimal consumption path is derived by following Blanchard[3], we first obtain the marginal utility with respect to debt and label it φ where

$$\varphi = \frac{U"(c)}{U'(c)}dc \tag{28}$$

By substituting (22) into (21) we solve this differential equation in the usual manner and obtain

$$c = c_0 e^{\frac{(\sigma+n-r)}{\varphi}} \tag{29}$$

Equation (13) aids us to determine the debt path, by substituting c from (29) and the per capita equivalent of (10) we obtain

$$\frac{d[\mathrm{d}]}{dt} = (1-\delta\sigma)f(k)(1-\Lambda\gamma(\hat{r})Y/\delta) - c_0 e^{\frac{(\sigma+n-r)}{\varphi}} - i(1-\rho i/k). \tag{30}$$

We solve for the optimal debt path with the usual integrating factor and noting that the debt must be paid off in a finite period, this results in

$$d = \int_0^T [1-\delta\sigma)f(k)(1-\Lambda\gamma(\hat{r})Y/\delta) + i(1-\rho i/k)]e^{(r-n)s} + c_0 e^{\frac{(\sigma+n-r)}{\varphi}-(r-n)s}ds + d_0 e^{(r-n)t}. \tag{31}$$

The next step in the analysis is to determine the intertemporal budget constraint , we facilitate this by ensuring that there is an explicit term for the initial debt stock

$$d_0 = \int_0^T [1-\delta\sigma)f(k)(1-\Lambda\gamma(\hat{r})Y/\delta)+i(1-\rho i/k)]e^{(r-n)s}+c_0 e^{\frac{(\sigma+n-r)}{\varphi}-(r-n)s}ds+d_0 e^{(r-n)t}-de^{(n-r)t}. \tag{32}$$

[3] Blanchard, Olivier and Stanley Fischer., 'Lectures on Macroeconomics'. M.I.T Press, Cambridge Massachusetts 1989

Intuitively, this constraint should tell us that consumption should be less than or equal to the representative agent's income and debt. We use the transversality conditions to show The dynamic of debt accumulation is linked to the behavior of consumption and its impact on the intertemporal constraint . The analysis of the transition state helps to shed light on this. Recall equation (29), simple manipulation yields

$$\frac{\dot{c}}{c} = \frac{(\sigma + n - r)}{\varphi} \tag{33}$$

How does this relationship between labor force growth ,interest rate , and time preference explain debt behavior ?

In equation (30) the optimal consumption is shown to be stable over time and determined by the initial wealth condition. For this to be true ,then it is clear that wealth must change over time to accommodate the optimal consumption remaining constant . We relate this to debt accumulation by integrating the portion of (31)) that addresses consumption

$$de^{(n-r)t} = d_0 \int_0^{\infty} \sum^{*} {}^{4} e^{(n-r)s} + c_0 e^{\frac{r-n-\sigma}{\varphi}(n-r)t} \frac{\varphi(c)}{r-n-\sigma} - \frac{c_0 \varphi}{r-n-\varphi} = n - r. \tag{34}$$

[4] $\sum^{*} = [1 - \delta\sigma] f(k)(1 - \gamma[\Lambda(r)/\delta)]) - i(1 + \rho i/k)$

3 Transition Dynamics

3.1 I

When consumption rises faster than $(r-n)$, we know that the intertemporal constraint is ineffective ,however since $(r-n) > \delta$ in $(\delta + n - r)$, our logical conclusion is that

$$\varphi(c) > 1 - \frac{\delta}{n-r} > 0. \tag{35}$$

From (26) and (28) we conclude that it is possible for this net debtor to become a net creditor since

$$\lim_{t\to\infty} d = -\infty \tag{36}$$

3.2 II

In this case we want to show that the economy's debt accumulation does not occur at the expense of consumption. This is so for the following reason, if $\delta + n - r > 0$ then from (31) we know that

$$\lim_{t\to\infty} e^{(n-r)t} d = d_0 + \int_t^{\infty} \sum^{*} e^{(n-r)s} - \left(\frac{r - \delta n}{\varphi(c)} - (r+n) \right)^{-1} c_0 = 0 \tag{37}$$

. We deduce from this that and that $\delta > r + n$.

The intertemporal constraint tells us that

$$\lim_{t\to\infty} d = 0 \tag{38}$$

$$Q.E.D$$

3.3 III

When $\delta + n - r = 0$ we achieve the same result as I. This can be shown by examining the limits of (31) i.e.

$$\lim_{t\to\infty} e^{(r-n)t} d = d_0 + \sum^{*} e^{(n-r)s} ds + \left(\frac{c_0}{(r-n)}\right) = 0$$

This result implies that the model economy moves from net indebtedness to become a net lender since

$$\lim_{t\to\infty} d = \frac{c_0}{-(r-n)} < 0 \tag{39}$$

4 The Steady State

By definition the full employment level of output,$\sum^*$ is constant in the steady state equilibrium. This can be used to provide a solution to (31) . If $\sum^e$ is the full employment level of output we write the solution as

$$de^{(n-r)t} = d_0 - \frac{\sum^e e^{(n-r)t}}{(n-r)} + \frac{\sum^e}{(n-r)} + \left(c_0 e^{\frac{-\delta-n-r}{\varphi(c)}-(n-r)t} - c_o\right)\frac{\varphi c_0}{r-\delta-n} + n - r \tag{40}$$

The results from the transition state are confirmed in the steady state by using equation (40)) in the same manner in which (31) was used in the transition analysis to analyse the conditions under which the intertemporal constraint is satisfied. The findings are summarised for different configuration of the determining parameters in the model

$$\delta + n - r \quad < \quad 0 \Rightarrow \varphi(c) > \frac{\delta}{r-n} \tag{41}$$

$$\delta + n - r \quad > \quad 0 \Rightarrow \varphi(c) > 0 > \frac{\delta}{r-n} \tag{42}$$

$$\delta + n - r \quad = \quad 0 \Rightarrow \varphi(c) > \frac{\delta}{r-n} \tag{43}$$

From (41) the model economy will move to a state of net creditorship since the limit of the debt equation is negative

$$\lim_{t\to\infty} d = -\left(\frac{\sum^e}{(r-n)}\right) = -\infty$$

(42) forces the conclusion that when these condition are binding the economy will remain

indebted

$$\lim_{t\to\infty} d = \left(\frac{\sum^e}{(r-n)}\right) > 0. \tag{44}$$

Equation (43) provides another clue to the possible path of debt in the steady state, if the initial stock of debt is positive, then the steady state will be akin to the outcome in *equation* (42).On the other hand if $d_o = 0$ then the economy's steady state is one in which debt is zero. We see this from

$$\lim_{t\to\infty} e^{-(r-n)t} d = d_o + \frac{\sum^e}{(r-n)} - \frac{c_0}{(n-r)} = 0 \tag{45}$$

The additional inference drawn earlier is understood (as well as its implication that a positive initial debt means perpetual indebtedness), since

$$\lim_{t\to\infty} d = \frac{\sum^e}{(r-n)} - \frac{c_0}{(n-r)} > 0.$$

4.1 Stability Analysis

I follow the literature in examining the system's stability when the fundamental equations are linearised. We first define θ as the composite shadow price of capital where

$$\theta = \frac{v}{\chi} \tag{46}$$

To derive an expression for the shadow price of capital in terms of the composite price, we use equation (25) to obtain

$$v = \chi\theta e^{(\delta-n+r)t}. \tag{47}$$

Since the steady state growth rate[5] and shadow price of capital are constant ,we can deduce

[5]Steady state values of variables are represented by letters with a bar.

that

$$\frac{\bar{i}}{\bar{k}} = \eta + n.$$

Similarly, equation (18) provides the insight that

$$\bar{\theta} = (1+\rho)(\eta+n) + \rho'(\eta+n). \tag{48}$$

The steady state value of the marginal product of capital can be obtained by substituting into (15) and differentiating. This gives

$$\bar{f}'(k) = \frac{(\eta+n)(1+\rho(\eta+n)) - (r-n)\bar{\theta}}{(1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])} \tag{49}$$

By manipulating (49) we obtain the steady state time derivative of the composite shadow price

$$\frac{\partial\bar{\theta}}{\partial t} = (\eta+n)\theta - [(1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])f(k) + (i/k)^2\rho(i/k)] \tag{50}$$

This tell us that the growth rate of the composite shadow price of capital is a function of the ratio of investment to the capital stock and the shadow price itself. For tractability we denote the ratio of investment to the capital stock as $\omega(\theta)$ and $\partial\bar{\theta}/\partial t$ as $\ddot{\theta}$, which allows us to rewrite (51) in the form

$$\ddot{\theta} = J(f(\theta), \theta, k, \omega(\theta))$$

The most important check for stability is undertaken by linearising the growth rate of the composite shadow price in the region of the steady state

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{k} \end{bmatrix} = \begin{bmatrix} 0 & \omega(\theta) \\ (1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])f"(k) & \theta + r \end{bmatrix}^{-1} \begin{bmatrix} \theta - \theta^{\alpha} \\ k - k^{\alpha} \end{bmatrix} \tag{51}$$

The system is saddle point stable with the determinant of the general matrix in equation (51) equal to $-(1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])f"(k)\omega(\theta)$.The roots of the systems are opposite signs

$$\lambda_{1,2} = \left(1/2 \right)^{-1} \left((\theta + r) \pm \sqrt{(\theta + r)^2 - 4(1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])f"(k)} \right).$$

4.2 Terms of Trade

The most important element in analysing the terms of trade conditions is the impact of variations in the price of the imported input. An increase its price reduces its own demand and that of domestic capital because these two goods are complements. The fall in demand for the imported input causes the shadow price of debt to rise relative to the shadow price of domestic capital. From equation (46), it is clear that the composite shadow price will decrease in response to the movement in the shadow price of debt. Since the capital - investment ratio is fixed, the lowering of the composite shadow price causes the equilibrium capital stock to fall when the adjustment process is finished. These interrelations can be shown by totally differentiating equation (49).

$$\begin{aligned}(1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])f"(k)dk &= \bar{\theta}d\bar{k} + (1-\delta\sigma)(1-\gamma[\Lambda(r)/\delta])f'(k)d\sigma + [(1-\delta\sigma)f'(k)/\sigma] \\ &\quad [\Lambda(r)d\gamma + \gamma d\Lambda(r)/\delta]d\delta \qquad (52)\end{aligned}$$

5 Conclusion

The simple model presented shows that the behavior of debt and consumption are related to the rates of growth of the labour force ,the discount rate and world interest rate. We show that an increase in the shadow price of debt causes the composite price to fall then eventually increase to bring the the capital stock to a lower magnitude . The impact of debt on the current account depends on whether the analysis is in the transition or steady state .

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