International Reserves, External Debt and Sovereign Bond Spreads

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ABSTRACT

Since the 1990s, emerging markets have accumulated substantial stocks of international reserves to act as buffers against external shocks. However, increasingly, several authors have emphasized a role for international reserves in reducing the probability of a sudden stop to capital inflows and the resulting roll-over risk of upcoming debt maturities. Yet, while some research has studied the role of foreign exchange reserves in reducing the marginal cost of borrowing (see Levy Yeyati, 2008 and Bianchi et al, forthcoming for example) and advocate some role for countries to borrow to 'top up' reserves for precautionary purposes and reduce default risk and sovereign risk premiums, most of these studies focus their attention on larger emerging markets who frequently tap international capital markets with little differentiation between small and large countries. In fact, stylized facts presented in this paper suggest that smaller countries pay higher spreads on sovereign debt than larger markets. Thus, this paper seeks to ascertain the role of international reserves in reducing the spread on external sovereign bonds and to determine whether that effect varies by country size. Leveraging data for 27 emerging markets, the paper finds evidence that growth in reserves reduces bond spreads, but at a decreasing rate as reserves levels get higher. Further, this relationship is weaker for small countries than it is for large ones. Thus, while the paper presents evidence that countries can borrow to accumulate reserves and subsequently reduce bond spreads, smaller countries have less capacity to do this than their larger counterparts.

KEYWORDS: Sovereign risk premium, International reserves, External Debt

INTRODUCTION

It has been well documented that, since the 1990s, emerging markets have accumulated substantial stocks of international reserves to act as buffers against external shocks. More generally, international reserves have several uses, with the International Monetary Fund (2013) noting that most managers of international reserves globally opt to hold reserves for precautionary motives, while several also maintain reserves to manage exchange rate fluctuations in line with monetary policy objectives. In summary, the research reviewed to date has highlighted a critical role for international reserves in reducing the cost of financial crises (see literature review for a more comprehensive discussion). Many authors have emphasized the importance of limiting sharp declines in output and subsequently absorption or consumption due to external shocks and support a role for reserves as a tool to minimise consumption volatility within a central bank's or government's loss function.

However, beyond the benefits of reserves post-external shock or a sudden stop in capital inflows, several authors also note a role for reserves in reducing the probability of a sudden stop in capital inflows and consequently in reducing the chance of a crisis. Jeanne and Ranciere (2011) and Kim (2017) are among those who include a role for reserves in restricting the chances of a sudden stop in their models of optimal reserve accumulation (see the literature review for a more comprehensive list of studies), while Qian and Steiner (2017) note that greater reserve holdings may reduce a sovereign's cost of borrowing, flatten its yield curve and encourage greater long-term borrowing. This increases the maturity of external debt, increases available foreign exchange buffers and consequently reduces roll-over risk. This benefit of holding more reserves implies a role for countries to tap international debt markets to build reserves buffers and protect against both the probability and effects of externally-driven shocks to the balance of payments. However, holding more debt leaves sovereigns susceptible to enhanced default risk and could further increase interest rate spreads and roll-over risk. Thus, debt and international reserves managers must strike a delicate balance between building reserve buffers and leaving the country exposed to additional risks.

Yet, while some research has studied the role of foreign exchange reserves in reducing the marginal cost of borrowing (see Levy Yeyati, 2008 and Bianchi et al, forthcoming for example), most of these studies focus their attention on larger emerging markets who frequently tap international capital markets with little differentiation between small and large countries. Levy Yeyati (2008) find that holding greater levels of reserves reduces bond spreads and this effect is even greater for markets with fixed exchange rates. He also illustrates that the cost of borrowing to build reserves is likely overstated given these favourable benefits to spreads. Similarly, leveraging data for Mexico, Bianchi et al (forthcoming) suggest that borrowing to accumulate international reserves may actually increase bond spreads if the favourable effects of greater reserves on debt does not offset the adverse effects of higher debt on spreads. However, while smaller markets tend to have regimes with more stable exchange rates (see Appendix A1), less focus is placed specifically on understanding how this phenomenon relates specifically to smaller, developing markets, many of whose external debt to GDP ratios exceed those of their larger counterparts, but who also depend heavily on financing from official creditors (usually at more favourable interest rates) relative to larger markets. This paper illustrates this stylized fact and seeks to provide initial evidence for whether country size constrains smaller markets from gaining appropriate access to global financial markets by way of a higher risk premium beyond that that may be expected with higher debt levels relative to reserve holdings. Specifically, the paper attempts to shed some preliminary light on the following questions:

1. Does issuing debt to accumulate international reserves reduce average bond spreads? If yes, above what level of reserves does this relationship fall away and does this relationship vary by country size?
2. Do smaller markets pay a higher premium on sovereign debt than their larger counterparts?

The paper contributes to the literature by building on work by Levy Yeyati (2008) and Bianchi et al (forthcoming) to test and identify varying non-linear relationships between bond spreads and international reserves and external debt, and to identify country-size specific thresholds above which borrowing to bolster international reserves holdings increases sovereign spreads and in turn roll-over risk.

Preliminary results confirm a common finding in the literature that international reserves may reduce the spreads on sovereign debt, while rising external debt also increases the sovereign risk premium that investors demand on foreign debt. Moreover, the potentially greater effect of reserves (compared to external debt) on spreads at relatively low levels of reserves reduces the risk premium (on a net basis) if the government opts to issue external debt to build foreign exchange buffers. However, as governments accumulate more reserves, the marginal benefit of these reserves to reducing a country's probability of default and ultimately sovereign risk premiums falls and for the sample of 27 countries in this study, borrowing to build reserves once reserves are already above 15% of GDP may increase bond spreads and by extension the risk of rolling over upcoming debt. The results also suggest that this relationship varies among countries, and smaller economies benefit less from a marginal increase in reserves levels than their larger counterparts. Thus, the threshold of reserves to GDP above which additional borrowing increases spreads is lower for smaller economies than larger ones.

The rest of the paper is structured as follows: Section 2 provides an overview of the literature on the role of international reserves accumulation for precautionary motives, while Section 3 highlights some stylized facts on the relationships between sovereign bond spreads and country size. Section 4 briefly summarizes the theoretical framework mapping reserves, debt and sovereign bond yields and presents the methodology and data used in the study. Section 5 presents and discusses the results, and Section 6 concludes with policy implications and considerations, and future areas of research.

LITERATURE REVIEW

A substantial body of literature has long documented the theoretical and empirical importance of international reserves accumulation by countries worldwide. The International Monetary Fund (2015) highlights several uses for international reserves, including to support a fixed exchange rate or transfer wealth across generations, among others. International reserves play a vital role in exchange rate management and monetary policy in emerging markets. Central banks in emerging markets worry about excessive exchange rate volatility and its cost to consumption and general economic activity. Calvo and Reinhart (2002) describe this as the "fear of floating" and suggest that many emerging market economies who have a strong commitment to inflation targeting and claim to have flexible exchange rates use monetary policy tools and foreign exchange reserves accumulation to limit the extent of exchange rate volatility (and subsequent pass-through to inflation) to an acceptable level. Ghosh, Ostry and Chamon (2015) suggest that "…emerging market economy central banks, including those with inflation-targeting frameworks, place a premium on exchange rate stability…." The authors advocate that foreign exchange market intervention and changes in interest rates should be used jointly to ensure that both objectives are met (Ghosh, Ostry and Chamon, 2015). Canova (2005) explains that the similarities in the response of output in floating exchange rate economies and non-floating rate economies in Latin America in response to economic shocks emanating from the USA may result from central banks in floating exchange rate economies using international reserves to minimise the volatility in their exchange rates. Aizenman and Riera-Crichton (2008) also illustrate that holding large buffers of foreign exchange reserves can reduce the effects of shocks to the terms of trade on exchange rates, while Aizenman et. al (2012) provide evidence that active management of reserves in response to shocks to the commodity terms of trade in Latin American economies can especially help to support weakening currencies.

Small, open economies also desire exchange rate stability to contain inflation and minimize its impact on consumption and overall welfare. Worrell et al. (2018) emphasize the importance of exchange rate stability to small, open, financially-integrated economies whose economies are characterized as having "…(a) high export concentration; (b) a limited range of competitive tradeable production, compared with import needs; and (c) a domestic financial system which is fully integrated into world financial markets..." (Worrell et al., 2018). Further, Worrell (2012) contends that, in a small, very open economy highly dependent on earnings of foreign exchange to facilitate imports for most of its domestic consumption, a stable exchange rate supports a low inflation target "…because it does not aggravate the effects of imported inflation." Active foreign exchange reserve management and accumulation, and minimising exchange rate volatility may therefore ultimately be essential to reducing the negative effects of currency depreciation and volatility on consumption and overall welfare (Bahmani-Oskooee et. al, 2015).

Notwithstanding the aforementioned uses of international reserves, the primary motives for accumulation have traditionally been segmented into two schools of thought – the mercantilist motive and the precautionary motive. Authors define the mercantilist motive as the byproduct of a policy to promote export competitiveness. Policymakers purchase foreign exchange and accumulate reserves to limit the degree of currency appreciation and encourage export-led economic growth (Aizenman and Lee, 2007). In contrast, the precautionary motive describes a deliberate strategy to build foreign exchange liquidity buffers as self-insurance in anticipation of external shocks (Aizenman and Lee, 2007). Bar-Ilan and Marion (2009) attempt to link the two motives to explain reserve accumulation in Asian economies as they believe that it makes little sense to separate the issues of reserve accumulation for insurance against shocks and output/inflation stabilization. They explain that reserve accumulation and exchange rate policy are linked, in that the level of international reserves affects the level of the exchange rate policymakers choose and the exchange rate in turn influences the reserves. Targeting the exchange rate permits the central bank to achieve output and inflation targets via export-led growth, while the subsequent accumulation of reserves reduces the probability of a financial crisis and the associated loss in output. Still, policymakers must balance the need to maintain a weak currency to boost economic growth and the political pressure which naturally arises from a perceived undervalued exchange rate (Bar-Ilan and Marion, 2009).

While there still appears some debate in the literature about which motive provides the dominant explanation for international reserves accumulation since the 1990s, several studies suggest a key role for the precautionary motive in most central banks over the last few decades. Aizenman and Lee (2007) tested the relative importance of each motive for reserves accumulation in fifty-three advanced, emerging and developing economies over the period 1980 to 2000 and found stronger evidence of the precautionary motive than the mercantilist motive in explaining rising international reserves over that period. Further, they noted that "…existing patterns of growing trade openness and greater exposure to financial shocks by emerging markets go a long way towards accounting for the observed hoarding of international reserves…" (Aizenman and Lee, 2007). Similarly, the International Monetary Fund (2013) highlights that over 70% of country authorities surveyed identified "precautionary liquidity needs" as the key reason for accumulating international reserves, while approximately 40% maintained reserves to manage the exchange rate. Bar-Ilan and Lederman (2007) and Kato et. al (2018) suggest that including international reserves as one of the central bank's target variables may permit it to reduce the probabilities that the economy experiences financial and currency crises respectively, while Shrestha and Semmler (2014) provide empirical evidence that suggests that, given their foreign exchange constraint and concerns about financial stability, central banks in five eastern and south-eastern Asian economies generally react more strongly to fluctuations in inflation and the international reserves than they do to the real effective exchange rate, the foreign interest rate and the output gap.

However, the structure of an economy plays a key role in determining the nature of the shocks each country is susceptible to and the extent to which a specific volume of international reserves is deemed adequate to insure against external shocks. For example, Moore and Glean (2016) employed a cost-benefit approach to estimate the appropriate level of reserve holdings for small states vulnerable to natural disasters and other external shocks to reduce output losses associated with a crisis. Considering that holding reserves also comes at an opportunity cost to policy makers, the authors estimated that, depending on the government's fiscal stance, the optimal level of reserve holdings could rise to as high as 25 weeks of imports, more than double that of the global rule of thumb of 12 weeks of imports. Most notably, the actual level of foreign reserves required depended on the structure and overall policy framework within those economies (Moore and Glean, 2016). Further, Crispolti's (2018) study of small states illustrated that "…the effectiveness of international reserves as a buffer against external shocks depends on the type of shock that is experienced as well as on the structural characteristics of the economy…" (Crispolti, 2018). In fact, he found that small states with fixed exchange rates tended to hold less reserve buffers than their floating-rate counterparts (Crispolti, 2018), a result somewhat consistent with Bar-Ilan and Marion (2009) who illustrated that commitment to a fixed exchange rate reduces the level of reserves required to protect against future crises. The International Monetary Fund (2015), in its third of three reports which guide the assessment of reserve adequacy in its member countries, focused on the need to hold precautionary reserves for three types of countries – mature (advanced), countries with global financial market access (typically emerging markets) and countries with limited global financial market access (typically low-income or developing countries). Mature or advanced economies tend to hold reserves to reduce the probability of foreign exchange shortages in the domestic economy, emerging economies with financial market access worry about mitigating crises emanating from current account or (more particularly) capital account shocks including currency crises and sudden stops to capital inflows, and low-income countries are concerned with protecting domestic absorption against shocks to the external current account (International Monetary Fund, 2015). While the study emphasized the use of the import coverage ratio as an appropriate method to determine low-income economies' resilience to current account shocks, the IMF proposed a revised reserve adequacy metric for emerging markets with financial access. This metric sought to ensure that emerging markets build adequate protection against:

1. Terms of trade shocks which may lead to volatile export revenues,
2. Potential capital flight by residents,
3. Roll-over risk of short-term, external debt, and
4. Other sudden stops or reversals in capital inflows, particularly from previously-built up liabilities.

The International Monetary Fund (2015) weighted each vulnerability to capture its relative importance to emerging markets and determined that roll-over risk and the risk of sudden stops from other capital account liabilities represented the greatest risks to this segment of economies. The recent rise in emerging markets' financial development and openness and their attractiveness as markets for foreign investment (Qian and Steiner, 2017), and the volatility of emerging market interest rates which move counter to the business cycle (Neumeyer and Perri, 2005) appear to validate this determination. Further, Worrell et. al (2018) noted that financial flows, and not trade, dominate the foreign exchange market in small, financially-integrated economies in the short-run.

Given the increased dependence of emerging markets on foreign debt funding, a sudden stop to capital inflows or the inability to roll-over upcoming, short-term maturities may increase the probability of sovereign default. This of course comes at a cost. Mendoza and Yue (2012) leveraged the Eaton-Gersovitz (1981) model of default to investigate the effects of sovereign default on external debt on countries' and ultimately firms' access to credit markets and the impact on production. The authors assumed that default increases the cost of firms' access to foreign working capital and forces them to substitute previously-imported intermediate inputs for domestic inputs which are imperfect substitutes. The latter's lower productivity leads to output loss for the firm and the country. Further, Na et al. (2018) posit that the decline in output associated with sovereign defaults may or may not lead to substantial declines in employment, but this depends on the nature of a country's exchange rate regime. Policymakers who are willing and able to adjust their exchange rate to prevent the surge in unemployment which may accompany the fall in output due to downward nominal wage rigidity, may devalue the domestic currency (by at least 35% in their study) to reduce the extent of real exchange rate overvaluation, reduce real wages and keep employment stable (Na et al., 2018). Alternatively, no currency adjustment led to a 20-percentage point rise in unemployment in their model. Ultimately, Mendoza and Yue's (2012) research implies that being more open or relying on external finance to fund imported inputs exacerbates the consequences to sudden stops or loss in capital market access. Thus, open economies without available domestic substitutes are likely to experience greater output loss at the time of default. These countries will default less (at higher debt levels) because they recognize the cost of doing so is much higher than less open counterparts (Mendoza and Yue, 2012). Further, Mendoza and Yue (2012) illustrated that exclusion from external markets reduces the capacity for the government to borrow to smooth consumption when output declines compared to borrowing during good times to finance greater consumption. However, throughout the study, the authors assume that the country does not accumulate foreign savings before a default to permit it to drawdown on those funds during the period when it is subsequently excluded from global credit markets. This implies that countries cannot build foreign exchange reserves to act as buffers during a crisis and runs counter to the trends witnessed in emerging markets since the 1990s but is likely a function of the Eaton-Gersovitz (1981) assumption that debt matures in one period (see also Bianchi et al., forthcoming for the implications of this assumption on required reserves holdings).

This susceptibility to external shocks and dependence on imports creates an excessive consumption volatility relative to output volatility (Kodama, 2013). Kodama (2013) illustrates that, notwithstanding access to global financial markets, since many developing economies depend heavily on imports for domestic consumption, shocks which disturb their ability to finance imports will directly impact consumption even more so than income. Further, the author points out that these economies typically suffer from "…a volatile terms of trade, a volatile borrowing interest rate, the acceptance of aid, and a monocultural economy…" (Kadoma, 2013). In the final analysis, Kodama (2013) illustrates that his Dynamic Stochastic General Equilibrium (DSGE) model which appropriately accounts for the characteristics of the small, open, low-income economy can explain 79% of the difference in consumption volatility between Kenya (his sample low-income economy) and Canada (his proxy for a larger, industrialised economy). Again however, like Mendoza and Yue (2012), the authors abstract from the presence of international reserves as potential buffers against external shocks.

Several studies have since incorporated a role for international reserves in providing a buffer against sudden stops and in reducing the probability of a sudden stop associated with investors choosing not to roll over short-term debt. Levy Yeyati (2008) emphasize the effects which greater reserves may have on interest rate spreads on existing debt. He illustrates empirically that, while reserve accumulation carries an opportunity cost (since the cost of borrowing or yield from foregone investment usually exceeds the yield from risk-free assets), greater reserve holdings actually reduce the spreads on existing debt, especially for fixed exchange rate economies (Levy Yeyati, 2008). In fact, he notes that the marginal cost of reserve accumulation may be overstated by over 50% if the effects on spreads are not accounted for (Levy Yeyati, 2008).

Jeanne and Ranciere (2011) develop a small, open economy model where consumers risk losing access to external borrowing markets and may choose to hold reserves relative to short-term debt to insure against losses in consumption arising from a sudden stop to capital inflows. Their model derives an expression for the level of reserves which maximises the consumer's welfare where reserve holdings are positively related to the likelihood, size and output cost of a sudden stop episode, and the risk aversion parameter, and negatively related to the cost of accumulating reserves (Jeanne and Ranciere, 2011). The authors find that their calibrated model can replicate the average level of reserves relative to GDP for Latin American economies, but they fail to reach the level of reserves accumulated by Asian economies (Jeanne and Ranciere, 2011). Solving the latter discrepancy requires an assumption of greater output costs arising from sudden stops and a significant rise in the risk aversion parameter, both of which may arise from the actual experience and lingering fears of East Asian economies coming out of the late-1990s financial crisis (Jeanne and Ranciere, 2011). However, it does not explain why China has accumulated the magnitude of reserves it has over the past two decades.

Jeanne and Ranciere (2011) also augment their insurance-against-sudden-stops model to allow the probability of a sudden stop to depend negatively on the reserves to short-term debt ratio to capture the role of reserves in displaying confidence in the economy. While they find that this additional benefit theoretically increases the optimal level of reserves desired by policymakers, Jeanne and Ranciere (2011) alternatively find no empirical evidence that international reserves reduce the probability of a crisis. Instead, the level of public indebtedness, degree of real exchange rate overvaluation and the degree of financial openness to foreign inflows materially affect a country's probability of experiencing a sudden stop in capital inflows (Jeanne and Ranciere, 2011). However, Prabheesh (2013) finds that higher reserve holdings reduce the probability of sovereign default, which in turn improves the country's credit rating, reduces the cost of borrowing and maintains access to international capital markets. Using India as an example, Prabheesh (2013) illustrates that the inverse of reserves as a ratio of short-term debt, the size of the government's fiscal deficit, and the "…volatility of foreign institutional investment…" (Prabheesh, 2013) are all significant determinants of that country's sovereign risk premium.

Hur and Kondo (2016) (as do other authors in this strand of literature) leverage the popular Diamond and Dybvig (1983) bank run model to present a framework for determining the optimal level of reserves relative to external debt in the presence of roll-over risk. In this model, investors borrow to finance investment in a technology which yields a certain level of output. However, if production is stalled, the investment may be liquidated, and yields returns less than the uninterrupted value of output. A sudden stop in capital inflows may force pre-mature liquidation and a less than desired reduction in output (Hur and Kondo, 2016). Thus, like in Jeanne and Ranciere (2011), international reserves carry a dual role in their model: they act as a liquidity buffer during a sudden stop but may also reduce investors' probability of not rolling over maturing debt (akin to the effects of deposit insurance on the likelihood of and in the presence of a bank run). Hur and Kondo (2016) endogenize reserve accumulation and the occurrence of sudden stops and permit governments to learn of liquidity shocks from each other. They find that global liquidity shocks may generate roll-over risks and produce substantially higher episodes of sudden stops (initially). Policymakers respond by increasing reserves, which reduces the probabilities of crises thereafter (Hur and Kondo, 2016). However, the slower policymakers learn about global liquidity shocks and the increased roll-over risk, the greater the likelihood that policymakers are underinvested in reserves and the greater the chances for sudden stops initially. Countries may in fact learn more slowly if their learning is restricted to the liquidity shocks occurring within their region and not necessarily shocks occurring globally (Hur and Kondo, 2016). Finally, Hur and Kondo (2016) suggest that, since each country's optimal response to higher roll-over risk is to hold more reserves, individual countries tend to hold more reserves than if they opted to pool reserves and share risks. This assumes of course that liquidity shocks to various countries are not perfectly, positively correlated. They note that the IMF could potentially opt as an option to provide liquidity in times of crisis and reduce the buildup of reserves, but the stigma associated with the IMF and their previous programmes may discourage some countries from relying on their assistance and hence promote overinvestment in foreign exchange reserves (Hur and Kondo, 2016).

While Hur and Kondo (2016) endogenize both reserves and sudden stops, they take the level of external debt as given. In fact, the results presented thus far would seem to imply that, to completely remove roll-over risk, countries should use excess reserves over debt to pay down outstanding liabilities. Further, the higher-risk nature of emerging markets' debt relative to the safe, liquid assets which comprise many countries' foreign reserve holdings imply an opportunity cost of reserve accumulation via debt. However, empirically, many countries globally choose to hold both debt and reserves (Kim, 2017), and oftentimes reserves levels exceed overall indebtedness. Kim (2017) also emphasizes that foreign exchange reserves help to reduce both the chances and costs of sudden stops to capital inflows, but their framework also attempts to jointly explain the ratios of foreign borrowing and external debt evidenced in emerging markets. While the model fails to individually explain the levels of reserves and external debt in the sample of developing countries (implying that countries accumulate reserves other than just for precautionary reasons), Kim (2017) points out that the assumption of limited enforcement and the inclusion of default risk are necessary to produce the result of joint holdings of reserves and debt (Kim, 2017). Holding both debt and reserves provides the option of default, which would otherwise not be available if governments repaid outstanding debt with reserves. The latter option is preferred if the cost of a sudden stop is too large, but otherwise, defaulting on debt during a sudden stop permits the government to "… transfer resources to default states…" (Kim, 2017) and use reserves to smooth consumption (Kim, 2017). However, Kim (2017) does note that while many authors assume an empirically-proven role for reserves in reducing the probability and costs associated with a sudden stop, "…their micro-foundation remains to be understood in future studies…" (Kim, 2017).

Most studies to date have also failed to appropriately account for the effects of maturity structure in models of reserve accumulation. Qian and Steiner (2017) investigate the effects of international reserves on a country's yield curve's term structure and ultimately the maturity of external debt. While reserves are taken as exogenous, countries' concern about current and future probabilities of experiencing financial crises prompt governments to hold reserves against these risks. However, reserves are found to both reduce and flatten the government's yield curve, thereby lowering the relative cost of issuing long-term external debt. Government's subsequent bias toward long-term borrowing to build more reserves increases the buffers available during a sudden stop and reduces the ratio of short-term debt relative to long-term debt. The dual effects on numerator and denominator reinforce the effects of reserves on financial stability as measured by the ratio of reserves to short-term debt (Qian and Steiner, 2017). However, quite interestingly, their Panel VAR and variance decomposition find virtually no evidence of the share of private and public long-term debt impacting the level of reserves (Qian and Steiner, 2017). Finally, as with Kim (2017), Qian and Steiner (2017) also suggest that countries hold both reserves and debt pre-default to later permit consumption smoothing when they have defaulted on external debt.

Additionally, Bianchi et al. (forthcoming) seek to determine the optimal level of foreign exchange reserves for emerging markets in the presence of rollover risk from the probability of a sudden stop. Although they do not formally model the government's debt maturity structures in their variant of the Eaton-Gersovitz (1981) model of sovereign default, their results depend on an assumption that the maturity of external debt exceeds one year (Bianchi et al., forthcoming). Bianchi et al. (forthcoming) highlight the tradeoff that governments face in using debt to accumulate reserves. In their model, a greater stock of reserves reduces roll-over risk, but (unlike in other studies) incurring more debt to build reserves actually increases the government's cost of borrowing. They find that the level of debt and reserves is increasing in the country's level of income but falling with creditors' aversion to risk. During high income periods, governments should incur debt and build reserves, but once negative income shocks occur, they use reserves to repay debt (Bianchi et al., forthcoming). Similarly, an increase in investors' risk aversion increases borrowing costs and reduces the incentive to borrow at higher rates. Governments therefore use reserves to meet debt maturities (Bianchi et al., forthcoming). Hence, borrowing to hold more reserves in good times shifts resources to tough times and reduces future consumption volatility. Finally, Bianchi et al.'s (forthcoming) sensitivity analysis highlights two key results:

1. If overall debt maturity is 1 year or less (in line with the maturity of the risk-free asset for reserve accumulation) the required level of reserves falls to almost zero and implies that governments do not benefit from lower rollover risk as a result of reserve accumulation if all debt matures in the same period as the reserve assets (Bianchi et al., forthcoming), and
2. Assuming a greater loss of income from default reduces required reserves (Bianchi et al., forthcoming) as the government will choose to hold less debt since the cost of defaulting on that debt is more prohibitive.

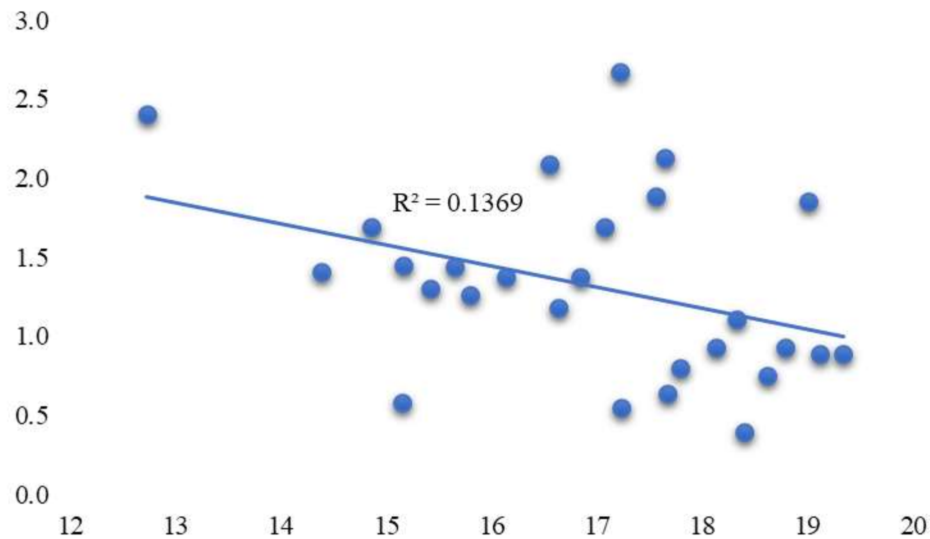
Finally, because countries default when reserves are already low and the paper's model doesn't account for post-default implications, Bianchi et al. (forthcoming) find that an increase in the length of time excluded from financial markets after default doesn't have a material impact on optimal reserve holdings. However, Aizenman and Sun (2012) suggest that uncertainty surrounding the duration of a crisis and the chances that, while reserve depletion today averts a crisis today, it increases the probability of future crises create a "fear of losing reserves". They documented that immediately upon the onset of the global financial crisis, some emerging markets initially allowed their reserves to fall in response to the crisis, primarily to pay short-term external debts. However, as the crisis continued, they slowed the depletion of reserves and instead allowed the exchange rate to adjust (Aizenman and Sun, 2012). Their short model with adjustment costs explains that central banks prefer to smooth the adjustment over multiple periods rather than respond aggressively initially and then lose substantial output/consumption in the latter periods. However, as Na et al. (2018) highlighted, the inability or unwillingness to use devaluation as an adjustment tool during a crisis could substantially increase domestic unemployment.

Notwithstanding the substantial literature reviewed to date, several questions and areas for advancement remain. For one, as Bianchi et al. (forthcoming), Qian and Steiner (2017) and others illustrate, paying closer attention to the maturity of external debt can substantially influence the recommended level of reserves countries may hold against rollover risk and sudden stops. Further, the role of fixed exchange rates (and by extension country size given the empirical relationship between the two) in determining whether economies should hold more or less reserves or can sustain more or less debt than 'floaters' remains unclear in the literature. Crispolti (2018) and Bar-Ilan and Marion (2009) imply empirically and theoretically respectively that emerging markets with fixed exchange rates may hold less reserves than emerging markets. However, the International Monetary Fund (2015) implies that lower-income countries without the flexibility of a floating rate to act as a shock absorber should hold more foreign exchange reserves, and they also apply greater weights to fixed exchange rate economies in their reserves adequacy metric for emerging markets with financial market access. Additionally, despite fewer empirical examples of sovereign defaults in fixed exchange rate economies, Na et al. (2018) indicate that, because default unlocks resources to aid in post-default economic recovery and limit the surge in unemployment associated with external shocks, these economies have a greater incentive to default and thus pay a higher spread on sovereign debt. Jahjah et al. (2013) also illustrate empirically that countries with fixed exchange rates pay higher bond spreads and issue less debt than their floating-rate counterparts. This reduces the level of sovereign debt that these economies hold relative to their floating-rate partners (assuming neither category of economy has access to bailout resources), but also helps to explain why their frequency of default is lower (Na et al., 2018). While this may suggest that fixed exchange rate economies require less reserves against their lower levels of debt, Levy Yeyati's (2008) earlier finding that fixed exchange rate economies benefit more from the impact of reserves on sovereign debt spreads and should therefore hold more reserves than their floating counterparts runs counter to this. Hence, the distinction between fixed and floating rate economies' (or small and large countries') determinations of optimal debt and reserve holdings remains a required area of investigation in this field of research.

STYLIZED FACTS

The charts and stylized facts in this section illustrate some of the motivating factors for the topic studied in this paper. Figure 1 below illustrates that smaller emerging markets pay higher spreads on their external debt than do their larger peers. A couple of potential reasons immediately come to mind. One thought is that smaller markets are more heavily indebted and/or hold substantially lower levels of international reserves than their larger counterparts, and thus, foreign creditors perceive them to have greater risk for this sole purpose. Another school of thought could be that, beyond smaller markets' debt and international reserves levels, private commercial investors believe that smaller markets are inherently riskier (as highlighted in Na et. al, 2018 and Jahjah et al, 2013).

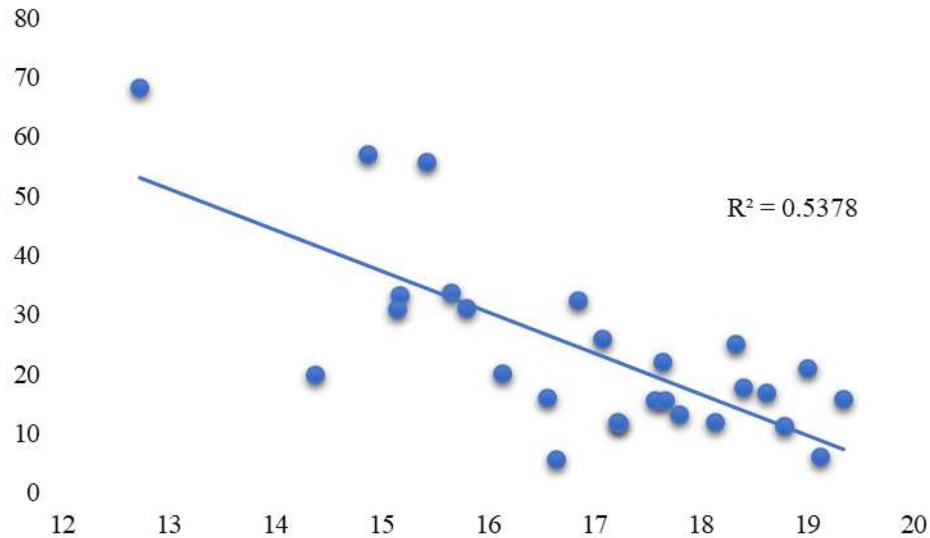
Figure 1: Cross-sectional Relationship between Average Natural Log of EMBIG Spreads (y-axis) and Average Ln(Population) (x-axis)



Source(s): JP Morgan, World Bank, author's calculations.

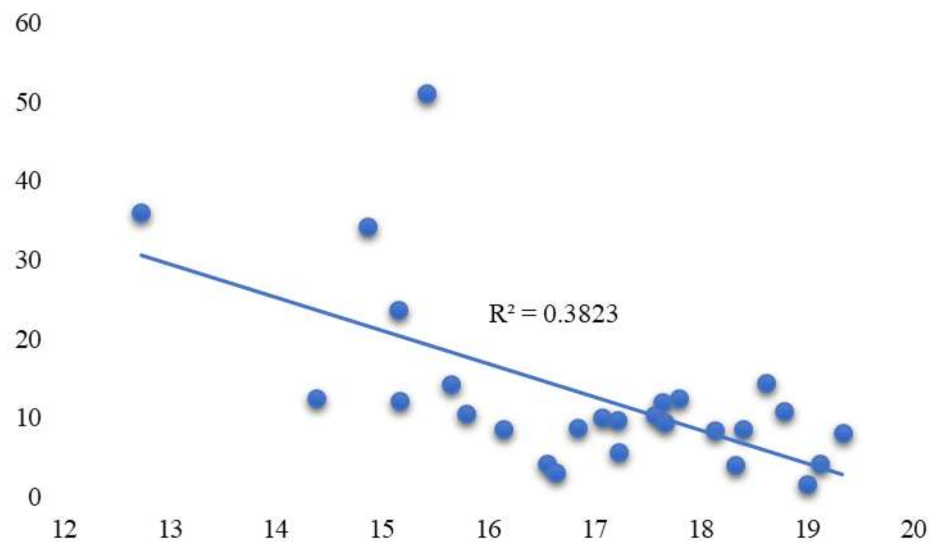
Figures 2, 3 and 4 attempt to tackle the first question. As highlighted by Kadoma (2013), smaller, open economies depend heavily on foreign financial inflows including sovereign borrowing to finance much of their consumption and investment needs. Inevitably, these small markets have accumulated substantial stocks of external debt over their history. Figures 2 and 3 clearly illustrate that, on average, smaller economies owed substantially higher stocks of total public external and total public, commercial external debt relative to GDP between 2008 and 2016. Further, figure 4 highlights that smaller economies did not necessarily hold more reserves than their larger counterparts. Of note, the bilateral R^2 between reserve holdings and population size for 27 emerging markets included in the sample is just 3%. However, the strength (or lack thereof) of this relationship masks a large outlier (Lebanon) in the data whose international reserves/GDP ratio averaged 80% over the sample period. Excluding Lebanon, the strength of the relationship falls to an even more paltry 0.5%. Thus, the relative difference between debt and reserves stocks for large and smaller economies may hypothetically help to explain the difference in sovereign bond spreads.

Figure 2: Cross-sectional Relationship between Average Public External Debt (% of GDP) (y-axis) and Ln(Population) (x-axis)



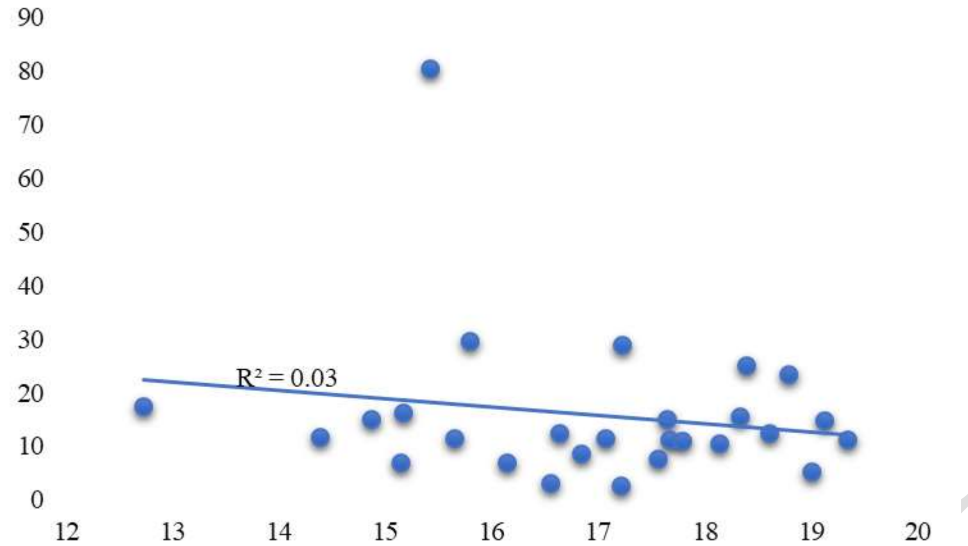
Source(s): World Bank, author's calculations.

Figure 3: Cross-sectional Relationship between Average Public Commercial External Debt (% of GDP) (y-axis) and Ln(Population) (x-axis)



Source(s): World Bank, author's calculations.

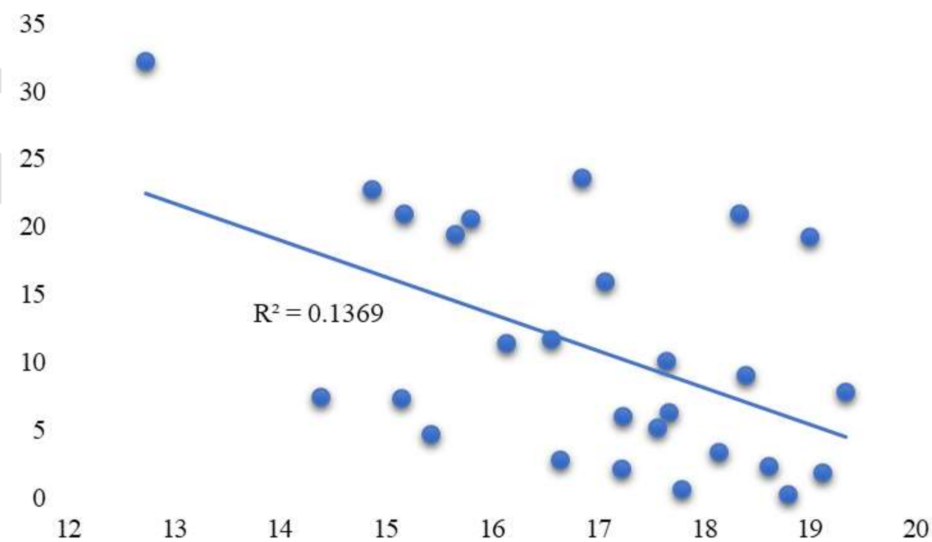
Figure 4: Cross-sectional Relationship between Average International Reserves Excluding Gold (% of GDP) (y-axis) and Ln(Population) (x-axis)



Source(s): International Monetary Fund, World Bank, author's calculations.

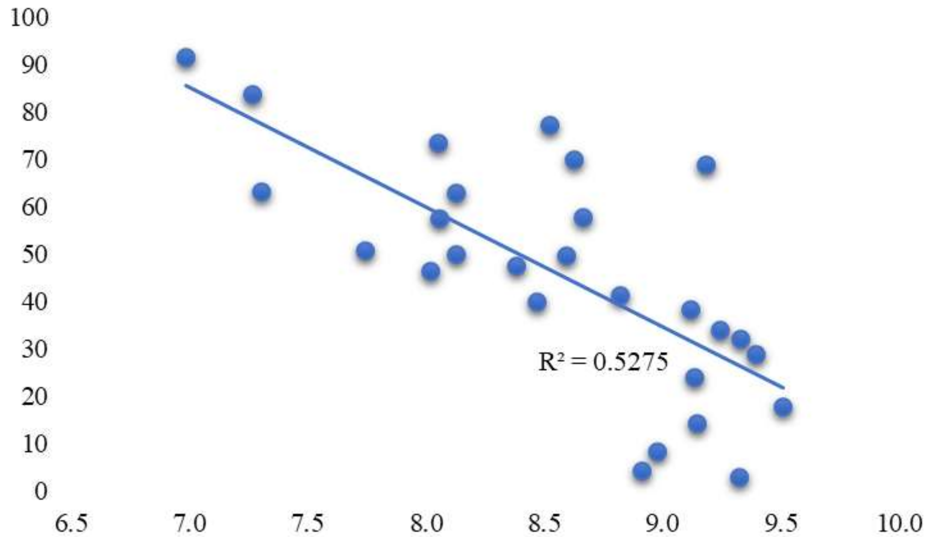
However, smaller and less developed markets also depend more heavily on official and cheaper sources of financing than their larger counterparts (see Figures 5 and 6). While the latter relationship is quite obvious (poorer countries should and do benefit from cheaper sources of funding and aid), the finding that smaller countries owe more official debt than larger countries requires further investigation.

Figure 5: Cross-sectional Relationship between Average Official Creditor Debt (% of GDP) (y-axis) and Ln(Population) (x-axis)



Source(s): World Bank, author's calculations.

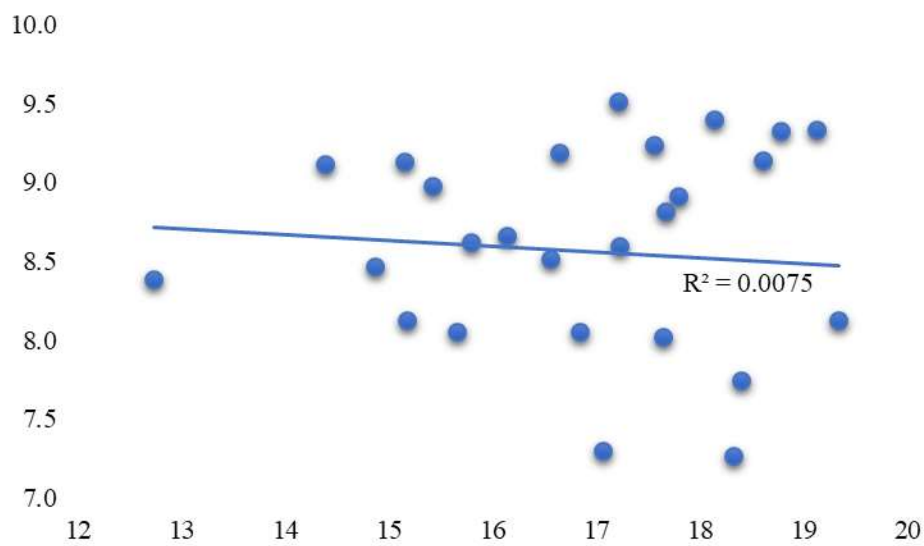
Figure 6: Cross-sectional Relationship between Average Official Creditor Debt (% of Public External Debt) (y-axis) and Average Ln(GDP per Capita) (x-axis)



Source(s): World Bank, author's calculations.

One potential school of thought may be that, among the sample of 27 countries chosen in this analysis, smaller countries may just happen to be the least developed. However, Figure 7 below suggests that this may not be the case as there appears almost no linear relationship between the natural log of population size and GDP per capita. Therefore, whether to finance development or for other reasons, smaller economies owe more debt to official creditors than their larger counterparts.

Figure 7: Cross-sectional Relationship between Average Ln(GDP per Capita) (y-axis) and Average Ln(Population) (x-axis)



Source(s): World Bank, author's calculations.

To sum up, it appears that smaller countries pay a higher risk premium on their commercial debt, and much of this is likely due to their stocks of debt and international reserves relative to larger peers. However, these economies also seem highly dependent on official sources of financing and this appears to have more to do than with just their level of development. While early to link this higher dependence on official financing to a higher risk premium, further analysis is required to determine whether this higher risk premium is due solely to higher levels of external debt relative to reserves, or due to other underlying factors.

METHODOLOGY AND DATA

Methodology

Bianchi et. al (forthcoming) articulate the relationships between a country's external debt, its stock of international reserves and sovereign spreads within a framework in which the government choose optimal levels of debt and reserves given investors' concern about default risk (see Appendix A2 for detailed derivations). The authors' analysis suggests that the net benefits of issuing debt to accumulate reserves depends on whether the favourable effects of higher reserves on bond spreads outweigh the adverse effects of greater debt holdings on those spreads. Similarly, Levy Yeyati (2008) show theoretically that the marginal cost of issuing debt to purchase reserves is a function of the current spread on debt, the responsiveness or spreads to greater reserves and debt respectively and the ratio of reserves to debt (see Appendix A3). Countries therefore need to consider both the stocks of debt and reserves when choosing to hold sovereign debt. Thus, there likely comes a point, above which, borrowing to hold reserves provides no additional benefits to the country. However, estimating this effect and threshold depends on the empirical specification of the relationship between bond spreads and stocks of international reserves and debt.

Having identified the theoretical relationships between spreads and reserves and debt, it is necessary to establish an appropriate empirical specification for the test equations to come based on a combination of previous work and graphical analysis. While Levy Yeyati (2008) do not assume the explicit form of the relationship between the risk premium and debt and reserves in their theoretical framework, he does assume a log-log relationship in both cases in his empirical specification.

The empirical models in this paper build on some of those derived in the literature on the determinants of sovereign bond spreads to date. Specifically, Tebaldi et al. (2018) leverage Edwards' (1984, 1986) framework to link the probability of sovereign default to a country's sovereign bond spread. In this framework, a risk-neutral lender's equilibrium position for a one period-bond is given by:

$$\left(1 + r_f\right) = (1 - p)\left[1 + \left(r_f + RP\right)\right] \quad (1)$$

where r_f represents the risk-free rate, RP captures the country's risk premium/spread on the one-period sovereign bond and p the probability of default that the sovereign debtor defaults on the entire debt and the investor recovers no funds. Solving for the risk premium, equation (1) can be represented as a linear function of the probability of default and the risk-free rate:

$$RP = \left[\frac{p}{(1-p)}\right]\left(1 + r_f\right) \tag{2}$$

Further, the authors assume that the probability of default follows a logistic function of the form $p = \frac{e^{\sum \beta X}}{1+e^{\sum \beta X}}$, where matrix X represents j economic fundamentals which determine the country's probability of default and β captures the relevant coefficients for each variable in X. Thus, equation (2) can be rewritten as:

$$RP_t = e^{\sum \beta X_t}\left(1 + r_{f_t}\right) \tag{3}$$

where the subscript *t* represents time. After taking natural logs throughout, adding the standard white noise error term ε_t and coefficients for a constant term (α) and the risk-free rate (γ), equation 4 below becomes the general form of regressions estimated for individual bond spreads in this study:

$$\ln RP_t = \alpha + \sum_{j=1}^{1} \beta_j X_{j,t} + \gamma \ln\left(1 + r_{f_t}\right) + \varepsilon_t \tag{4}$$

Therefore, for each country, the risk premium can be estimated as a function of macroeconomic fundamentals and the relevant risk-free rate.

Data

Specifically, this paper examines the determinants of sovereign bond spreads for 27 emerging markets using annual data over the period 2009 – 2016 (see Appendix A4 for a complete list of countries and related average population sizes). The period, while short and determined based on data availability at this time, captures economic and financial developments across numerous markets since the advent of the global financial crisis including the fall and subsequent rise of the global economy and the volatility in global energy prices which has substantially influenced the levels of external debt and reserves holdings in both commodity importers and exporters.

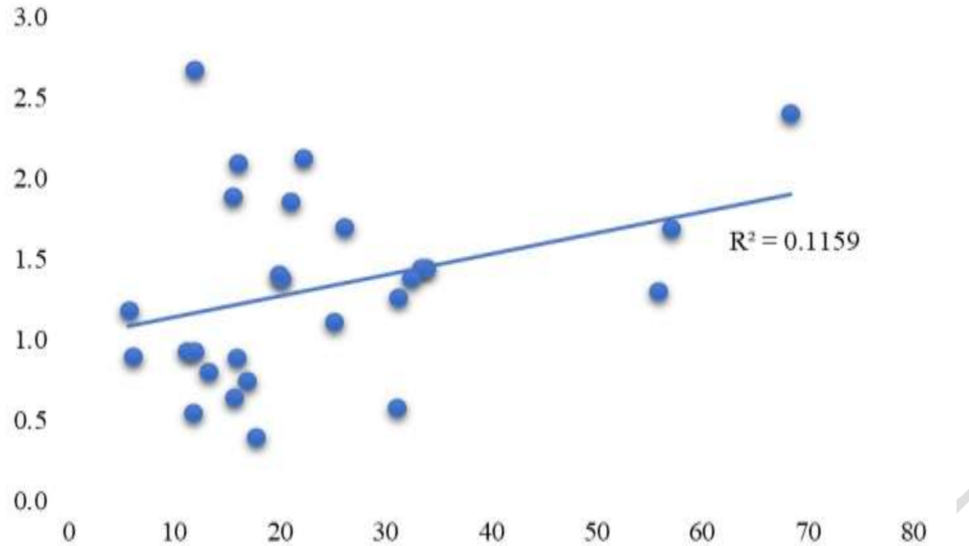
Sovereign spreads RP_t for each country are captured by and derived from JP Morgan's EMBIG index, while the yield on US 10-year Treasury bonds proxies the risk-free interest rate. Further, for the choice of regressors and given the relatively short time series of the data, the paper leverages some of the regressors captured in Tebaldi et al. (2018) and Prabheesh (2013). International reserves as a ratio of nominal GDP (Res_t) captures the level of international buffers available to the country and is expected to reduce a country's risk profile and its borrowing costs, while external government debt/nominal GDP (Ext_t) measures the overall indebtedness of the country relative to foreign creditors and a higher level of indebtedness implies a greater probability of default, ceteris paribus. In this case, external debt holdings are alternatively proxied by total public external debt (including concessional debt) and public and publicly guaranteed external debt issued to private creditors. A similar latter measure (one excluding concessional debt) is used by Ley Yeyati (2008) in his analysis of bond spreads and changes in this measure are likely to be more closely related to fluctuations in spreads than changes in total external debt holdings. Real per capita GDP (GDP_t) represents economic strength and is expected to reduce bond spreads, while the CBOE Volatility Index (VIX_t) measures expected volatility of the S&P 500 index and proxies overall global risk aversion. Greater risk aversion likely increases sovereign bond spreads for emerging markets. Population size (Pop_t) proxies the size of the country and is included to test whether smaller markets pay higher or lower premiums on sovereign debt. Finally, while Tebaldi et al. (2018) suggest roles for other political and economic determinants of sovereign bond spreads in their panel framework, the short nature of the time series in this paper and the likely resulting stability of many of these variables suggested that they may be less relevant to this analysis.

Sovereign bond spreads are all sourced from JP Morgan, international reserves (excluding gold) are sourced from the International Monetary Fund's International Financial Statistics, total public external debt (including concessional debt), public and publicly guaranteed external debt issued to private creditors, population size, GDP per capita at constant prices and GDP at current US prices are sourced from the World Bank, while the CBOE Volatility Index and US 10-year Treasury yield are captured from the St. Louis Federal Reserve Economic Data (FRED).

Equation 4 above implies that spreads are a non-linear function (and the natural log of spreads are a linear function) of all macroeconomic fundamentals, while figure 5 of Bianchi et. al (forthcoming) confirms the non-linear relationship between bond spreads and debt to income levels with various fixed values for income and lenders' risk aversion. The analysis suggests that as debt levels increase, spreads rise, and the slope of this relationship increases at an (apparently) exponential rate as debt levels rise.

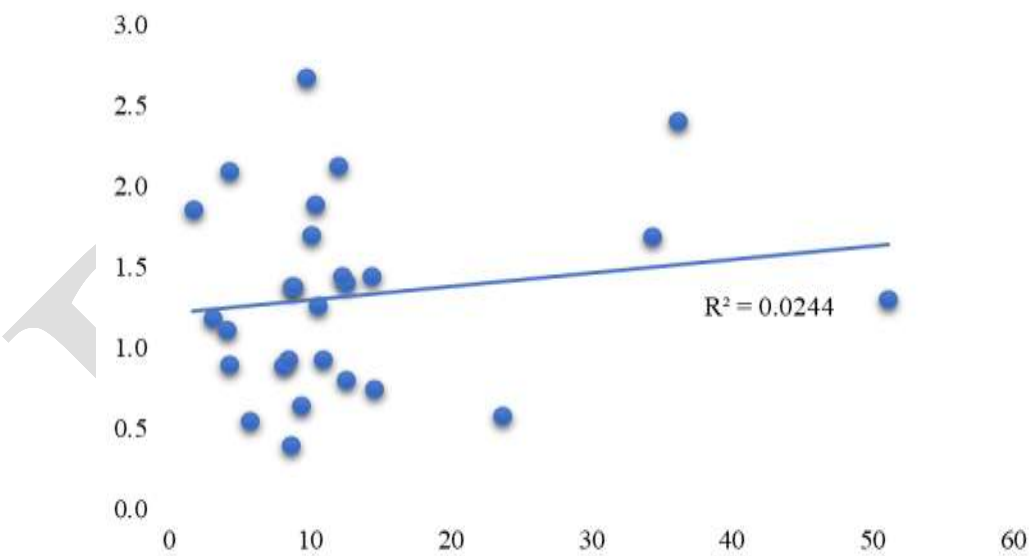
This paper's own analysis appears to confirm that view. Figure 8 below suggests a linear (and hence exponential) relationship between the natural log of spreads (hence actual spreads) and a country's ratio of public external debt to GDP. A similar (albeit slightly weaker) bivariate relationship holds between the natural log of spreads and the ratio of public and publicly guaranteed (PPG) external debt issued to private creditors as a ratio of nominal GDP (see figure 9). The latter relationship is key as it excludes credit issued by lenders at concessional terms, the cost of which is usually much lower than the spreads paid in commercial borrowing markets.

Figure 8: Cross-sectional Relationship between Average Natural Log of EMBIG Spreads (y-axis) and Total Public External Debt/GDP (x-axis)



Source(s): JP Morgan, World Bank, author's calculations.

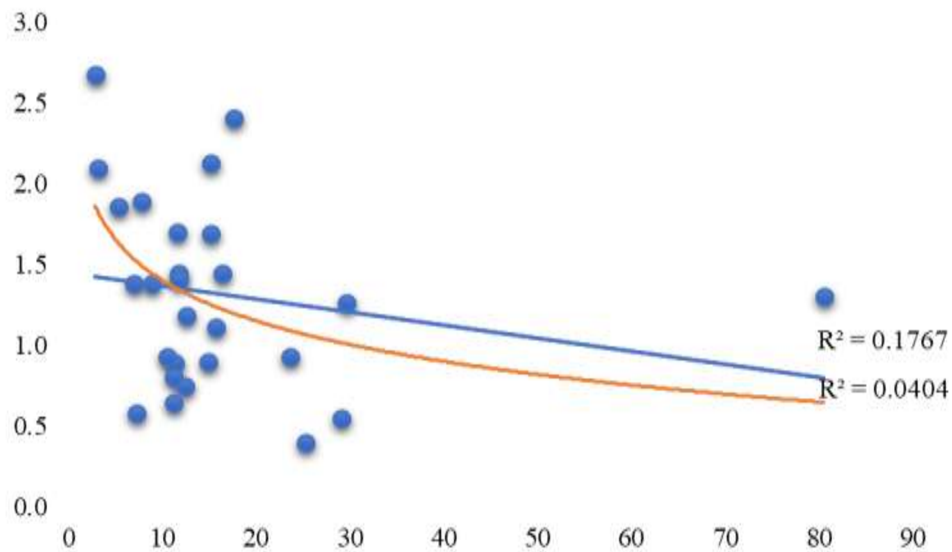
Figure 9: Cross-sectional Relationship between Average Natural Log of EMBIG Spreads (y-axis) and Public and Publicly Guaranteed (PPG) External Debt Issued to Private Creditors/GDP (x-axis)



Source(s): JP Morgan, World Bank, author's calculations.

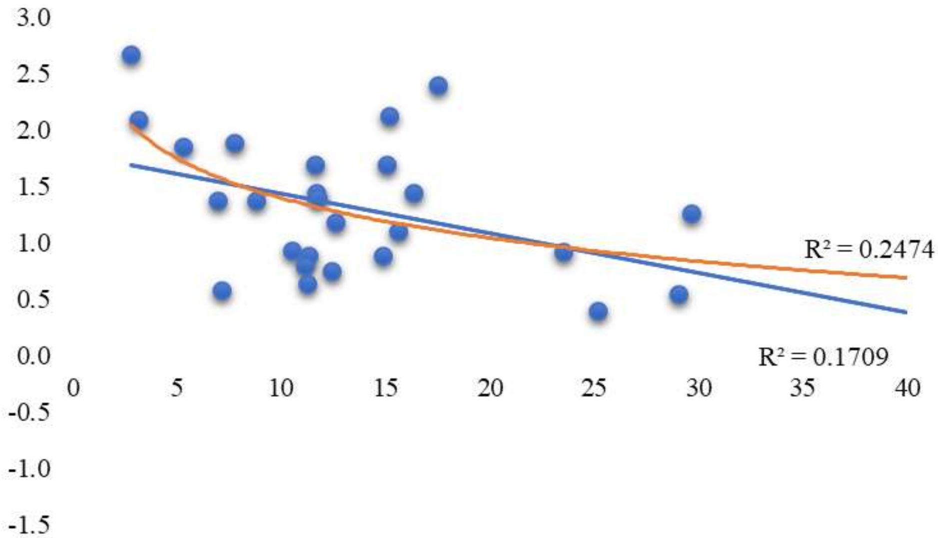
The relatively modest linear relationships highlighted in the figures immediately above neglect to account for the mitigating role which the stock of international reserves plays in reducing sovereign bond spreads. Figures 10 and 11 below illustrate the materially negative relationship between the two variables, but with a twist. In the previous two charts, a linear trendline could approximate the relationship between ln(spreads) and the respective debt variables, but visual plots appear to suggest that countries with larger stocks of international reserves generally enjoy lower spreads, but at a decreasing rate as proxied by a logarithmic trendline. This finding is supported by the graphical estimates provided by Bianchi et. al (forthcoming) in Figure 9 of their paper. The authors illustrate that, as a country borrows to build its stock of foreign exchange reserves, the (in their case) rise in spreads becomes more pronounced with higher levels of debt and reserves. This suggests that, as both debt and reserves levels rise, the marginal benefit of higher reserves on spreads is unable to keep pace with the negative fallout of higher debt on spreads. These graphical findings inform the choice of model specification in the empirical section to follow and in this way divert from Levy Yeyati (2008) and Tebaldi et al. (2018).

Figure 10: Cross-sectional Relationship between Average Natural Log of EMBIG Spreads (y-axis) and International Reserves/GDP (x-axis) with Lebanon



Source(s): International Monetary Fund, JP Morgan, World Bank, author's calculations.

Figure 11: Cross-sectional Relationship between Average Natural Log of EMBIG Spreads (y-axis) and International Reserves/GDP (x-axis) without Lebanon



International Monetary Fund, JP Morgan, World Bank, author's calculations.

REGRESSION RESULTS

Given the persistence typically evident in emerging markets' bond spreads, versions of equation 4 are estimated using the Arellano Bover Dynamic Panel estimator to determine the effects of each regressor on sovereign bond spreads.

$$\ln RP_{i,t} = \beta_{i0} + \beta_1 \ln RP_{i,t-1} + \beta_2\, ln(GDP)_{i,t} + \beta_3 \ln Res_{i,t} + \beta_4 Ext_{i,t} + \beta_5 ln(Pop)_{i,t} + \beta_6 VIX_t + \gamma \ln\left(1 + r_{f_t}\right) + \varepsilon_t \quad (5)$$

It is important to note that typically, country specific variables would enter each regression with at least one lag as most news on economic indicators are likely to be released and absorbed by investors within a quarter, while the VIX_t affects the risk premium in the same period. Foreign investors and those who trade sovereign bonds will likely learn of new country specific data with a lag. For example, real GDP growth for the previous quarter is usually not available until at least a month after the end of the quarter, while news of international reserves levels also comes with a lag, albeit usually shorter. In contrast, stock market volatility and yields on risk-free assets are observable in real time. However, in this case, the use of annual data means that a lag of one period equates to an entire year between the realization of macroeconomic developments and investors reacting to news of these indicators. This length of lag is likely impractical and thus, given the short time series element of the dataset, the paper assumes that all variables have a contemporaneous impact on bond spreads.

Before estimating each linear regression, it is prudent to test whether each variable entering each regression has stationary properties or not. The presence of stationarity permits us to estimate the regression as specified in equation 5, but the presence of non-stationary variables requires one of two treatments: either (1) test for cointegration to determine whether long-run relationships exist among the variables or (2) difference each variable until it achieves stationarity. The latter option is chosen as the short time series (just seven years) hardly justifies the definition of the long-run. Table 2 below therefore presents Im-Pesaran-Shin unit root test results for each variable included in the regression analysis. For each country, all variables are integrated of order 0 [I (0)] at the 10% level of statistical significance except real GDP per capita and both indicators of external debt which are all I (1). In most instances, these latter three are differenced once before entering their respective regressions, but given that 7 years is hardly a long enough time with which to conduct appropriate unit root tests, the external debt indicators also enter alternative regression specifications in their level form.

Table 2: Results of Im-Pesaran- Shin Unit Root Tests

Variable	**Levels**		**1st Difference**	
	Test Statistic	*P-Value*	*Test Statistic*	*P-Value*
$\ln \boldsymbol{RP}_{i,t}$	-2.072	0.019**	n.a.	n.a.
$\boldsymbol{ln}(\boldsymbol{GDP})_{i,t}$	-1.013	0.156	-7.123	0.000***
$\ln \boldsymbol{Res}_{i,t}$	-2.813	0.003***	n.a.	n.a.
$Total\ Ext_{i,t}$	1.424	0.923	-8.221	0.000***
$Commercial\ Ext_{i,t}$	2.289	0.989	-5.277	0.000***
$\boldsymbol{ln}(\boldsymbol{Pop})_{i,t}$	-46.766	0.000***	n.a.	n.a.
$\boldsymbol{VIX}_t$	-29.342	0.000***	n.a.	n.a.
$\ln\left(\mathbf{1} + \boldsymbol{r}_{f_t}\right)$	-9.143	0.000***	n.a.	n.a.

*N.B. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; n.a. stands for not applicable*

Tables 3 and 4 below present Arellano-Bover GMM estimates of equation 5 with debt measured as public and publicly guaranteed external debt issued to private creditors and total external public sector debt respectively. Model 1a and 2a estimate a version of equation 5 without population size while models 1b and 2b add population size to that equation and estimate equation 5 directly. Despite unit root tests suggesting that both debt/GDP measures are non-stationary, models 1c and 2c replace $\Delta Ext_{i,t}$ with $Ext_{i,t}$ to maintain consistency with $\ln Res_{i,t}$ in levels and to test the robustness of the finding that $Ext_{i,t}$ actually possesses a unit root. Finally, models 1d and 2d estimate equation 5, but replace $\ln Res_{i,t}$ with $Res_{i,t}$.

Generally, the explanatory variables included in each model appear to carry their expected signs. Further, robust standard errors correct for potential serial correlation and heteroscedasticity while each model appears to satisfy the AR(1) and AR(2) tests. Specifically, while the test rejects the null hypothesis of no AR(1) correlation, it fails to reject that hypothesis for AR(2) correlation at all conventional levels of significance.

The estimates produced generally confirm that greater international reserves reduce sovereign risk premiums. Real GDP per capita growth carries the expected negative and statistically significant coefficient, while the volatility in US equity markets – a proxy for global risk aversion – has a positive and statistically significant impact on fluctuations in sovereign spreads in all regressions. Further, although the risk-free rate has a statistically insignificant impact on sovereign spreads, estimates from models 1b, 1d, 2b and 2d suggest that larger countries enjoy lower risk premiums than their smaller counterparts. Finally, some evidence exists to suggest that growth in public external debt (both commercial and total debt) increases sovereign spreads. However, this result appears to be larger and more robust when public and publicly guaranteed external debt issued to private creditors is used as the proxy for external indebtedness rather than when total public external debt is considered. This result does not appear surprising as commercial investors are likely more sensitive to a sovereign's indebtedness to other private bondholders who charge market rates to hold emerging markets' debt rather than total external debt which often includes obligations with very favourable interest rates and repayment agreements.

Table 3: Arellano-Bover Linear Dynamic Panel Regression Estimates of Equation 5: Public and Publicly Guaranteed External Debt issued to Private Creditors

Variable	Model 1a	Model 1b	Model 1c	Model 1d
$\ln RP_{i,t-1}$	0.208**	0.081	0.062	0.121
se	*0.088*	*0.105*	*0.113*	*0.107*
$\ln Res_{i,t}$	-0.341***	-0.407***	-0.439***	
se	*0.109*	*0.120*	*0.129*	
$\Delta Ext_{i,t}$	0.018	0.031**		0.035***
se	*0.015*	*0.013*		*0.013*
$Ext_{i,t}$			0.029***	
se			*0.011*	
VIX_t	0.052***	0.043***	0.046***	0.040***
se	*0.006*	*0.009*	*0.012*	*0.010*
$\mathbf{ln}\left(1 + r_{f_t}\right)$	0.263	0.170	0.162	0.216
se	*0.240*	*0.247*	*0.268*	*0.239*
$\Delta ln(GDP)_{i,t}$	-3.216**	-3.767***	-3.404***	-3.606**
se	*1.339*	*1.227*	*1.146*	*1.466*
$ln(Pop)_{i,t}$		-0.242***	-0.063	-0.284***
se		*0.079*	*0.109*	*0.103*
$Res_{i,t}$				-0.034*
se				*0.019*
Constant	0.739	5.429***	2.101	5.637***
se	*0.473*	*1.809*	*2.516*	*2.054*
AR(1) z stat	-3.029***	-3.526***	-3.537***	-3.609***
p-value	*0.003*	*0.000*	*0.000*	*0.000*
AR(2) z stat	0.561	0.350	-0.085	1.320
p-value	*0.575*	*0.726*	*0.932*	*0.187*
Observations	189	189	189	189

*N.B. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation*

Table 4: Arellano-Bover Linear Dynamic Panel Regression Estimates of Equation 5: Total Public External Debt

Variable	Model 2a	Model 2b	Model 2c	Model 2d
$\ln RP_{i,t-1}$	0.191**	0.056	0.061	0.089
se	*0.083*	*0.102*	*0.105*	*0.103*
$\ln Res_{i,t}$	-0.335***	-0.392***	-0.417***	
se	*0.110*	*0.118*	*0.125*	
$\Delta Ext_{i,t}$	0.006	0.015		0.018**
se	*0.012*	*0.010*		*0.009*
$Ext_{i,t}$			0.011	
se			*0.007*	
VIX_t	0.051***	0.042***	0.044***	0.039***
se	*0.006*	*0.009*	*0.011*	*0.010*
$\ln\left(1+r_{f_t}\right)$	0.239	0.131	0.134	0.170
se	*0.231*	*0.245*	*0.256*	*0.240*
$\Delta ln(GDP)_{i,t}$	-3.549***	-4.047***	-4.156***	-3.864***
se	*1.170*	*1.136*	*1.044*	*1.360*
$ln(Pop)_{i,t}$		-0.237***	-0.084	-0.279***
se		*0.076*	*0.135*	*0.102*
$Res_{i,t}$				-0.033*
se				*0.018*
Constant	0.809*	5.426***	2.595	5.653***
se	*0.460*	*1.739*	*2.977*	*2.025*
AR(1) z stat	-3.013***	-3.435***	-3.494***	-3.516***
p-value	*0.003*	*0.001*	*0.001*	*0.000*
AR(2) z stat	0.547	0.325	0.083	1.263
p-value	*0.585*	*0.745*	*0.934*	*0.207*
Observations	189	189	189	189

*N.B. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation*

However, the results produced thus far beg the question of whether Bianchi et al's (forthcoming) finding that issuing debt to hold more reserves increases spreads, and if so, above what level of debt or reserves does this relationship commence. To determine this requires us to find the total derivative of equation 5 with respect to an issuance of external debt, assuming that one unit of debt is issued to purchase one unit of reserves, or $\frac{\partial Res_{i,t}}{\partial Ext_{i,t}} = 1$. Thus, we have:

$$\frac{d \ln RP_{i,t}}{dExt_{i,t}} = \frac{\partial \ln RP_{i,t}}{\partial Res_{i,t}} \frac{\partial Res_{i,t}}{\partial Ext_{i,t}} + \frac{\partial \ln RP_{i,t}}{\partial Ext_{i,t}} \frac{\partial Ext_{i,t}}{\partial Ext_{i,t}}$$

Given that the model specification with public and publicly guaranteed external debt issued to private creditors as the proxy for external indebtedness yields more statistically significant and robust results, and given that the specification with $Ext_{i,t}$ yielded estimates not too dissimilar to those with

$\Delta Ext_{i,t}$, model 1c's estimates are used to derive the level of reserves above which issuing more commercial external debt will increase bond spreads and ultimately the cost of rolling over upcoming debt when it comes due. From model 1c:

$$\frac{\partial \ln RP_{i,t}}{\partial Res_{i,t}} = \frac{-0.439}{Res_{i,t}}$$

$$\frac{\partial \ln RP_{i,t}}{\partial Ext_{i,t}} = 0.029$$

$$\frac{d \ln RP_{i,t}}{dExt_{i,t}} = \frac{-0.439}{Res_{i,t}} + 0.029$$

Therefore, with higher levels of international reserves, every additional increase in debt which adds to the stock of reserves either increases sovereign spreads or reduces spreads by a smaller magnitude than before. To then determine the level of reserves above which increases in debt to build reserves increases the sovereign risk premium, we set $\frac{d \ln RP_{i,t}}{dExt_{i,t}} = 0$ and solve for $Res_{i,t}$. This yields:

$$\frac{d \ln RP_{i,t}}{dExt_{i,t}} = \frac{-0.439}{Res_{i,t}} + 0.029 = 0$$

$$Res_{i,t} = \frac{0.439}{0.029} = 15$$

Thus, for the sample of 27 emerging markets, borrowing commercially to finance reserves once international reserves exceed 15% of GDP increases sovereign bond spreads, and by extension enhances the perceived probability of default and roll-over risks.

Given the results highlighted in Tables 2 and 3 which suggest that smaller countries pay higher spreads on sovereign debt and a threshold exists above which borrowing to hoard reserves increases bond spreads, one question which may arise is, does the effect of reserves on spreads vary by country size? Specifically, do smaller countries benefit or suffer more from building reserve buffers?

To investigate this question, equation 5 is altered to interact an additional population size term with the level of international reserves. Simplifying, this yields equation 6. Having determined that spreads fall with greater levels of reserves, a negative coefficient for $\ln Res_{i,t} \times ln(Pop)_{i,t}$ would suggest that smaller countries benefit less from increases in reserves than larger countries.

$$\ln RP_{i,t} = \beta_{i0} + \beta_1 \ln RP_{i,t-1} + \beta_2\, ln(GDP)_{i,t} + \beta_7 \ln Res_{i,t} \times ln(Pop)_{i,t} + \beta_4 Ext_{i,t} + \beta_5 ln(Pop)_{i,t} + \beta_6 VIX_t + \gamma \ln\left(1 + r_{f_t}\right) + \varepsilon_t \quad (6)$$

Table 5 below illustrates estimates for equation 6 with measures for Public and Publicly Guaranteed External Debt issued to Private Creditors now denoted as $Priv\ Debt_{i,t}$ while total public external debt is denoted as

$Tot\ Ext_{i,t}$. The results suggest that growth in debt owed to private creditors, both in levels and growth rates increases sovereign spreads in a statistically significant manner, but changes in total external debt have no significant impact on spreads. Further, similar to estimates provided in Tables 3 and 4, market volatility increases spreads, while larger countries pay lower spreads, but this effect is statistically significant only in regressions with debt indicators included as first differences rather than in levels. Most notably, the coefficient on $\ln Res_{i,t} \times ln(Pop)_{i,t}$ is statistically significant and negative in each equation with similar magnitudes in all cases. This suggests that larger countries benefit more from incremental accumulation of international reserves, and similarly, smaller economies benefit less.

Table 5: Arellano-Bover Linear Dynamic Panel Regression Estimates of Equation 6: Interaction Between Reserves and Population Size

Variable	Model 3a	Model 3b	Model 3c	Model 3d
$\ln RP_{i,t-1}$	0.083	0.065	0.058	0.064
se	*0.105*	*0.113*	*0.101*	*0.104*
$\ln Res_{i,t} \times ln(Pop)_{i,t}$	-0.023***	-0.025***	-0.022***	-0.024***
se	*0.007*	*0.007*	*0.007*	*0.007*
$\Delta Priv\ Debt_{i,t}$	0.030**			
se	*0.013*			
$Priv\ Debt_{i,t}$		0.029***		
se		*0.011*		
VIX_t	0.044***	0.047***	0.042***	0.045***
se	*0.009*	*0.011*	*0.009*	*0.011*
$\ln\left(1 + r_{f_t}\right)$	0.171	0.166	0.133	0.137
se	*0.249*	*0.270*	*0.245*	*0.256*
$\Delta ln(GDP)_{i,t}$	-3.845***	-3.431***	-4.132***	-4.213***
se	*1.204*	*1.133*	*1.119*	*1.031*
$ln(Pop)_{i,t}$	-0.168**	0.016	-0.165**	-0.015
se	*0.070*	*0.102*	*0.068*	*0.132*
$\Delta Tot\ Ext_{i,t}$			0.014	
se			*0.010*	
$Tot\ Ext_{i,t}$				0.011
se				*0.008*
Constant	4.123***	0.711	4.156***	1.384
se	*1.590*	*2.342*	*1.529*	*2.885*
AR(1) z stat	-3.538***	-3.534***	-3.447***	-3.496***
p-value	*0.000*	*0.000*	*0.001*	*0.001*
AR(2) z stat	0.307	-0.154	0.290	0.043
p-value	*0.759*	*0.877*	*0.772*	*0.965*
Observations	189	189	189	189

*N.B. *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation*

With these new results, recalculating the threshold of reserves above which borrowing to 'top up' reserves increases spreads yields a relationship which is a function of population size. Again, given that the model specification with public and publicly guaranteed external debt issued to private creditors as the proxy for external indebtedness yields more statistically significant and robust results, and given that the specification with $Priv\ Debt_{i,t}$ yield estimates not too dissimilar to those with $\Delta Priv\ Debt_{i,t}$, model 3b's estimates are used to derive the level of reserves above which issuing more commercial external debt will increase bond spreads. From model 3b:

$$\frac{\partial \ln RP_{i,t}}{\partial Res_{i,t}} = \frac{-0.025}{Res_{i,t}} ln(Pop)_{i,t}$$

$$\frac{\partial \ln RP_{i,t}}{\partial Priv\ Debt_{i,t}} = 0.029$$

$$\frac{d \ln RP_{i,t}}{dPriv\ Debt_{i,t}} = \frac{-0.025}{Res_{i,t}} ln(Pop)_{i,t} + 0.029$$

Therefore, with higher levels of international reserves, every additional increase in debt which adds to the stock of reserves either increases sovereign spreads or reduces spreads by a smaller magnitude than before, but this effect now varies with the size of the population. To then determine the level of reserves above which increases in debt to build reserves increases the sovereign risk premium, we set $\frac{d \ln RP_{i,t}}{dPriv\ Debt_{i,t}} = 0$ and solve for $Res_{i,t}$. This yields:

$$\frac{d \ln RP_{i,t}}{dPriv\ Debt_{i,t}} = \frac{-0.025}{Res_{i,t}} ln(Pop)_{i,t} + 0.029 = 0$$

$$Res_{i,t} = \frac{0.025}{0.029} ln(Pop)_{i,t}$$

If we take the average population sizes of the largest and smallest populations in the sample to understand the contrasting effects of borrowing to finance reserves, we have:

$$Res_{Belize,t} = \frac{0.025}{0.029} \times 12.7 = 11$$

$$Res_{Indonesia,t} = \frac{0.025}{0.029} \times 19.3 = 17$$

Thus, for the smallest country in the sample of 27 emerging markets (Belize), borrowing commercially to finance reserves once international reserves exceed 11% of GDP increases sovereign bond spreads, and by extension enhances the perceived probability of default and roll-over risks, while the largest country, Indonesia can borrow to finance reserves up to reserves of 17% of GDP before investors charge higher spreads on new investments.

Finally, the results presented in this section provide some evidence that while greater accumulation of external debt increases sovereign spreads, growth in international reserves aids in reducing the marginal cost of borrowing. In fact, the results imply that borrowing externally to build a country's stock of international reserves may not be as costly as some may expect, since the net effect may be a lowering in that sovereign's yield curve and a reduction in sovereign roll-over risk. These results support Levy Yeyati (2008)'s finding that many studies have overestimated the marginal cost of reserve accumulation, and the findings give empirical support to the assumption that international reserves reduce, not only the cost, but the probability of a sudden stop to capital inflows. However, the results suggest that borrowing to purchase reserves becomes less valuable as reserves levels improve and thus sovereign debt managers should be careful about utilizing this medium to build reserve buffers. This effect varies with the size of the country, with larger countries being able to accumulate greater reserve levels before spreads begin to increase than their smaller counterparts. Lastly and somewhat related, the results provide some partial evidence that, even after controlling for relative stocks of international reserves and external debt, smaller emerging markets suffer from a higher risk premium and may thus either not be able to issue as much commercial external debt as their larger counterparts, must hold larger stocks of international reserves to mitigate the effects of a higher risk premium, or resort to holding greater levels of concessional debt to finance their development. Preliminary analysis suggests that, after controlling for GDP per capita, the latter of the three is probably most prevalent. These results also partially support those by Na et. al (2018) and Jahjah et al. (2013) who suggest that while fixed exchange rate economies (who are typically smaller) exhibit fewer instances of sovereign default, they have a greater incentive to default and thus pay higher premiums. However, the results somewhat run counter to those of Levy Yeyati (2008) who find that fixed exchange rate economies' sovereign spreads benefit more from building reserves buffers. The contrast with this latter finding suggests that, notwithstanding the high correlation between exchange rate flexibility and country size, investigating the individual relationships between reserves and debt and sovereign spreads by country size and exchange rate regime requires special and separate attention.

CONCLUSION

International reserves accumulation has become a common feature of the macroeconomic policy framework of many emerging markets worldwide as governments and central banks seek to protect consumption or manage exchange rate fluctuations in the face of external (and sometimes domestic) shocks. However, with emerging markets increasing their financial exposure to global capital markets, several authors (see Kim, 2017 as an example) now propose a role for international reserves accumulation in preventing sudden stops in capital inflows, reducing roll-over risks and curtailing the chance of a full-blown crisis. Further, recent research has suggested that the absolute value of the effects of reserves on sovereign risk premiums exceeds that of higher debt levels on sovereign risk premiums and implies a role for greater debt issuance to finance international reserves accumulation and simultaneously reduce a country's marginal cost of borrowing. However, to date, empirical work has paid little attention to the level of reserves above which borrowing to finance reserves yields little additional benefit and to what extent this relationship varies by country size. Further, previous empirical work suggests that markets with fixed exchange rate regimes (many of whom are smaller countries) pay higher spreads on their sovereign debt.

Therefore, this paper sought to answer the follow questions in the context of 27 emerging market economies:

1. Does issuing debt to accumulate international reserves reduce average bond spreads? If yes, above what level of reserves does this relationship fall away and does this relationship vary by country size?

2. Do smaller markets pay a higher premium on sovereign debt than their larger counterparts?

Initial results imply that growth in reserves does lower spreads on external debt and implies a role for borrowing to 'top up' international reserves which may lead to a net reduction in the overall risk premium, lower the marginal cost of borrowing and reduce roll-over risk. However, the benefits of this exercise fade as reserves accumulation rises and are even reversed as reserves levels exceed 15% of GDP. Moreover, the empirical analysis also seems to suggest that smaller markets pay a higher risk premium on external commercial debt and that the threshold for reserves above which borrowing to accumulate reserves increases sovereign spreads is lower for smaller countries. This should imply a lower debt-carrying capacity than their larger counterparts, but their larger external debt levels and reliance on official financing suggests that this has not hampered their appetite to acquire debt. Overall however, small economies should consider their level of international reserves and relative size when determining whether they will issue new debt for precautionary purposes.

Despite supportive results thus far, the analysis does suffer from some limitations. First, the analysis captures a period of just seven years post-crisis and may be representative of the specific events which occurred during that time rather than the general relationship among reserves, external debt and sovereign spreads. Secondly, this preliminary analysis leveraged annual data on sovereign bond spreads and other macroeconomic and financial indicators in the absence of high frequency debt data for some of the markets studied. Clearly, investors in financial markets, particularly the most liquid ones, react rapidly to changes in global, regional and domestic economic fundamentals and annual time series will likely not capture these rich dynamics. Thirdly, while the data captured 27 emerging markets, this sample is likely not large enough to draw inferences about emerging markets in general, especially the very smallest developing countries, many of whom were not included in the analysis.

Finally, given the limitations cited above, future work should focus on extending the sample of investigation to a longer period, with higher frequency data where available and an even larger group of countries. Doing such may permit more robust and conclusive results but may also permit an analysis of whether the effects of international reserves on marginal borrowing costs continue to vary by exchange rate regime as found by Levy Yeyati (2008) a decade ago and may shed further light on why smaller economies seem to pay a premium on sovereign debt. Finally, while the author still recognizes that building reserves buffers via export-led growth and persistent current account surpluses is likely the preferred method of international reserves accumulation for small open economies, future work should also focus on investigating whether a level of debt above which debt accumulation for reserve buffers increases spreads exists and whether that likely threshold varies by type of country.

Bibliography

Aizenman, J., & Lee, J. (2007). International Reserves: Precautionary Versus Mercantilist Views, Theory and Evidence. *Open Economies Review, 18*, 191–214.

Aizenman, J., & Riera-Crichton, D. (2008, November). Real Exchange Rate and International Reserves in an Era of Growing Financial and Trade Integration. *The Review of Economics and Statistics, 90*(4), 812-815.

Aizenman, J., & Sun, Y. (2012). The financial crisis and sizable international reserves depletion: From 'fear of floating' to the 'fear of losing international reserves'? *International Review of Economics and Finance, 24*, 250–269. doi:10.1016/j.iref.2012.03.004

Aizenman, J., Edwards, S., & Riera-Crichton, D. (2012). Adjustment patterns to commodity terms of trade shocks: The role of exchange rate and international reserves policies. *Journal of International Money and Finance, 31*, 1990–2016.

Bahmani-Oskooee, M., Kutan, A. M., & Xi, D. (2015). Does exchange rate volatility hurt domestic consumption? Evidence from emerging economies. *International Economics, 144*, 53–65.

Bar-Ilan, A., & Lederman, D. (2007). International reserves and monetary policy. *Economics Letters, 97*, 170-178.

Bar-Ilan, A., & Marion, N. P. (2009). A Macroeconomic Perspective on Reserve Accumulation. *Review of International Economics, 17*(4), 802–823.

Bianchi, J., Hatchondo, J. C., & Martinez, L. (Forthcoming). International Reserves and Rollover Risk. *The American Economic Review*, 1-47.

Calvo, G. A., & Reinhart, C. M. (2002, May). Fear of Floating. *The Quarterly Journal of Economics, 117*(2), 379-408.

Canova, F. (2005). The Transmission of US Shocks to Latin America. *Journal of Applied Econometrics, 20*, 229–251.

Diamond, D. W., & Dybvig, P. H. (1983, June). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy, 91*(3), 401-419.

Eaton, J., & Gersovitz, M. (1981, April). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies, 48*(2), 289-309.

Edwards, S. (1984, September). LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80. *The American Economic Review, 74*(4), 726-734.

Edwards, S. (1986, June). The pricing of bonds and bank loans in international markets: An empirical analysis of developing countries' foreign borrowing. *European Economic Review, 30*(3), 565-589.

Ghosh, A., Ostry, J., & Chamon, M. (2015). Two targets, two instruments: Monetary and exchange rate policies in emerging market economies. *Journal of International Money and Finance*, 1-25. doi:http://dx.doi.org/10.1016/j.jimonfin.2015.03.005

Hur, S., & Kondo, I. O. (2016). A theory of rollover risk, sudden stops, and foreign reserves. *Journal of International Economics, 103*, 44–63.

Ilzetzki, E., Reinhart, C. M., & Rogoff, K. S. (2017). Exchange Arrangements Entering the 21st Century: Which Anchor Will Hold? *NBER Working Paper No. 23134*, 1-85.

International Monetary Fund. (2013). *Assessing Reserve Adequacy - Further Considerations.* Washington, D.C.: International Monetary Fund.

International Monetary Fund. (2015). *Assessing Reserve Adequacy - Specific Proposals.* Washington, D.C.: International Monetary Fund.

Jahjah, S., Wei, B., & Yue, V. Z. (2013, October). Exchange Rate Policy and Sovereign Bond Spreads in Developing Countries. *Journal of Money, Credit and Banking, 45*(7), 1275-1300.

Jeanne, O., & Rancière, R. (2011, September). The optimal level of international reserves for emerging market countries: a new formula and some applications. *The Economic Journal, 121*, 905-930.

Kato, M., Proaño, C. R., & Semmler, W. (2018). Does international-reserves targeting decrease the vulnerability to capital flights? *Research in International Business and Finance, 44*, 64–75.

Kim, Y. J. (2017). Sudden stops, limited enforcement, and optimal reserves. *International Review of Economics and Finance, 51*, 273-282.

Kodama, M. (2013, September). External Shocks and High Volatility in Consumption in Low-Income Countries. *The Developing Economies, 51*(3), 278-302.

Levy Yeyati, E. (2008). The cost of reserves. *Economics Letters, 100*, 39–42.

Mendoza, E. G., & Yue, V. Z. (2012). A General Equilibrium Model of Sovereign Default and Business Cycles. *The Quarterly Journal of Economics, 127*, 889–946. doi:10.1093/qje/qjs009

Moore, W., & Glean, A. (2016). Foreign exchange reserve adequacy and exogenous shocks. *Applied Economics, 48*(6), 490-501. doi:10.1080/00036846.2015.1083085

Na, S., Schmitt-Grohé, S., Uribe, M., & Yue, V. (2018). The Twin Ds: Optimal Default and Devaluation. *The American Economic Review*, *108*(7), 1773–1819. doi:https://doi.org/10.1257/aer.20141558

Neumeyer, P. A., & Perri, F. (2005). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics, 52,* 345–380.

Prabheesh, K. (2013). Optimum international reserves and sovereign risk: Evidence from India. *Journal of Asian Economics, 28,* 76-86.

Prakash, K., & Maiti, D. (2016). Does devaluation improve trade balance in small island economies? The case of Fiji. *Economic Modelling*, *55*, 382–393.

Qian, X., & Steiner, A. (2017). International reserves and the maturity of external debt. *Journal of International Money and Finance, 73*, 399–418.

Shrestha, P., & Semmler, W. (2014). Monetary policy and international reserves: empirical evidence from East Asian countries. *International Journal of Finance & Economics, DOI: 10.1002/ijfe.1509*.

Tebaldi, E., Nguyen, H., & Zuluaga, J. (2018). Determinants of emerging markets' financial health: A panel data study of sovereign bond spreads. *Research in International Business and Finance, 45*, 82-93.

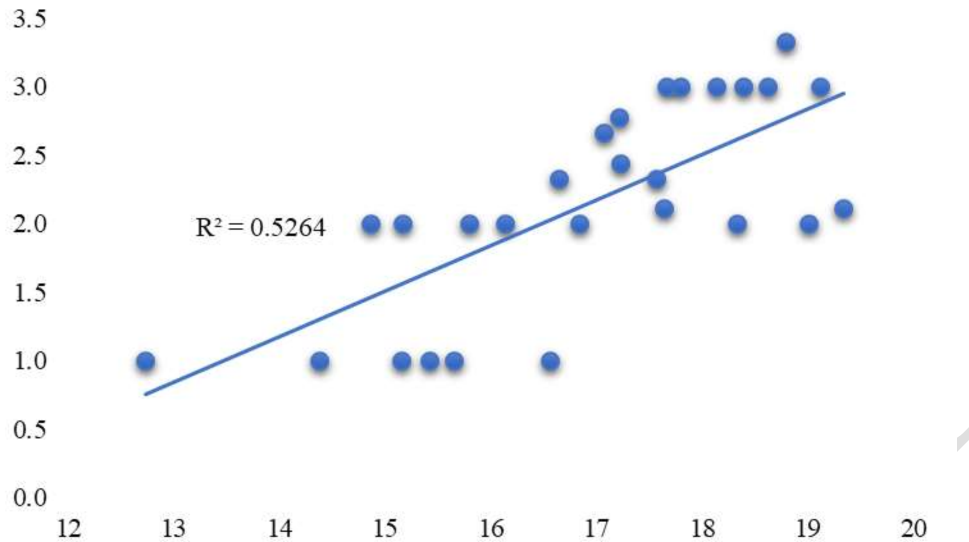
Valerio, C. (2018). Do small states need large external buffers? Evidence from an event-study analysis. In L. Briguglio (Ed.), *Handbook of Small States: Economic, Social and Environmental Issues (Europa Perspectives: Emerging Economies).* Taylor and Francis.

Worrell, D. (2012). Policies for Stabilization and Growth in Small Very Open Economies. *Group of Thirty, Washington, DC Occasional Paper 85*.

Worrell, D., Moore, W., & Beckles, J. (2018). A new approach to exchange rate management in small open financially integrated economies. In L. Briguglio (Ed.), *Handbook of Small States: Economic, Social and Environmental Issues (Europa Perspectives: Emerging Economies).* Taylor and Francis.

APPENDIX A1

Figure 7: Cross-sectional Relationship between Average Coarse Measure of Exchange Rate Flexibility (y-axis) and Average Ln(Population) (x-axis)



Ilzetzki et al. (2017), World Bank, author's calculations.

The chart above illustrates the relationship between a country's average estimated degree of exchange rate flexibility as measured by Ilzetzki, Reinhart and Rogoff (2017) and the average natural log of the country's population size. Ilzetzki, Reinhart and Rogoff's (2017) measure ranges from 1 to 6, higher values indicating a more flexible exchange rate regime. The chart clearly implies that larger economies tend to have more flexible exchange rate regimes and vice versa. While the relationship is far from perfect, the R^2 suggests a relatively strong relationship between the two variables.

APPENDIX A2: THEORETICAL FRAMEWORK FROM BIANCHI ET AL. (FORTHCOMING)

Bianchi et. al (forthcoming) articulate the relationships between a country's external debt, its stock of international reserves and sovereign spreads within a framework in which the government chooses optimal levels of debt and reserves given investors' concern about default risk. The authors define an economy whose endowment (y_t) is given by: $log(y_t) = (1-\rho)\mu + \rho log(y_{t-1}) + \varepsilon_t$, while the government's preferences over private consumption (c) are denoted as:

$$\mathrm{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j)$$

where β captures the appropriate discount rate.

Further, the government's debt (b_t) evolves as follows:

$$b_{t+1} = (1-\delta)b_t + i_t \tag{1}$$

with i_t = number of bonds issued in period *t*.

When the government has access to borrowing markets, its budget faces the following constraint:

$$c_t + g + \delta b_t + q_a a_{t+1} = y_t + a_t + i_t q_t \tag{2}$$

With δ = exogenous rate of decline of bond or coupon payments, q_t = price of bonds issued at period *t*, q_a = constant price of reserve assets and a_t = quantity of reserve assets (1-year asset) held at the beginning of period *t*. The reserve asset pays 1 unit of each consumption good, and so $a_t \geq 0$. Therefore, $q_a a_{t+1}$ = reserve accumulation and $i_t q_t$ = new issuances of debt.

In other words, households' consumption, the government's fixed expenditure (g), coupon payments on debt and the value of reserve accumulation are financed via total income, the starting level of reserves and new debt issuances during the period.

On the flip side, when the government loses access to capital markets via its decision to default, its budget constraint collapses and simplifies to:

$$c_t + g + q_a a_{t+1} = y_t + a_t$$

$$c_t = y_t + a_t - q_a a_{t+1} - g$$

Thus, consumption is financed solely from the domestic and foreign resources left back after the government makes its rigid and fixed outlays and accumulates additional reserves.

Investors price cashflows from sovereign bonds using the stochastic discount factor defined as:

$$m_{t,t+1} = e^{-r-(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma_\varepsilon^2)} \quad (3)$$

where r is the discount factor for foreign lenders and represents the risk-free rate, while κ_t is the parameter governing the risk premium shock with $\kappa_t \geq 0$. $\kappa_L = 0$ is the value of the shock in good times, while $\kappa_H > 0$ is the value of the shock in bad times. π_{LH} and π_{HL} are the corresponding transition probabilities for the risk premium shock which follows a two-state Markov process. Finally, ε_t captures income shocks where $\varepsilon_t > 0$ is a positive shock to income.

The authors note that the time-varying risk premium produced from the above will "…be endogenous to the gross portfolio positions chosen by the government, which determine default risk…." (Bianchi et. al, forthcoming). In other words, the government's choices of their stocks of debt and reserves produce a risk premium which varies over time and reflects the country's probability of default.

The government's optimization problem is therefore defined as the maximum value of the payoffs between repaying its debts and defaulting on its debts:

$$V(a,b,s) = max\{V^R(a,b,s), V^D(a,s)\}$$

where $V^R(a,b,s)$ = value to the government of repaying debts, $V^D(a,s)$ = value to the government of defaulting on debts and s = the current exogenous state of the world where $s = \{y, \kappa\}$.

Bianchi et. al (forthcoming) highlight that "…for any bond price function *q*, the function *V* satisfies…" the above equation. Obvious, yet important to note is that the government cannot borrowing during the default period.

The value of repaying foreign debt today is determined by:

$$V^R(a,b,s) = \underbrace{max}_{c\geq 0, a'\geq 0, b'\geq 0} \left\{u(c) + \beta \mathrm{E}_{s'|s} V^R(a',b',s')\right\}$$

Subject to:

$$c = y - \delta b + a + q(a',b',s')[b' - (1-\delta)b] - q_a a' - g$$

Where ($'$) denotes the next period value of that variable.

From equation (2) above,

$$c_t + g + \delta b_t + q_a a_{t+1} = y_t + a_t + i_t q_t$$

$$c_t = y_t - \delta b_t + a_t - g - q_a a_{t+1} + i_t q_t$$

Therefore, the value of bonds issued are equivalent to:

$$i_t q_t = q(a',b',s')[b' - (1-\delta)b]$$

since from equation (1),

$$i_t = [b_{t+1} - (1-\delta)b_t]$$

Further,

$$q_t = \ q(a', b', s')$$

which is the pricing function for new debt which depends on the level of reserves, debt and the state of the world.

Similarly, the value of defaulting on debt today is determined by:

$$V^D(a,s) = \underbrace{max}_{c\geq 0, a'\geq 0} \left\{u(c) - U^D(y) + \beta \mathrm{E}_{s'|s} V^D(a', 0, s')\right\}$$

subject to:

$$c = y + a + -q_a a' - g$$

Bianchi et. al (forthcoming) suggests that the solutions to this problem yield the following decision rules:

1. $\hat{d}(a,b,s)$: default
2. $\hat{b}(a,b,s)$: debt
3. $\hat{a}^D(a,s)$: reserves in default
4. $\hat{a}^R(a,b,s)$: reserves when not in default
5. $\hat{c}^D(a,s)$: consumption in default
6. $\hat{c}^R(a,b,s)$: consumption when not in default

Consistency with lenders' portfolio conditions necessitates that the bond price schedule satisfies the following equation:

$$q(a',b',s) = \mathrm{E}_{s'|s}\big[m(s',s)\big[1 - \hat{d}(a',b',s')\big][\delta + (1-\delta)q(a'',b'',s')]\big] \qquad (4)$$

Where:

$$b'' = \hat{b}(a',b',s')$$

$$a'' = \hat{a}^R(a',b',s')$$

In the above equation, $1 - \hat{d}(a',b',s') = 0$ if the sovereign has defaulted, while $1 - \hat{d}(a',b',s') = 1$ if the sovereign has not defaulted.

Therefore, if $\hat{d}(a',b',s') = 1$, then $q(a',b',s) = 0$ and the bond (under this framework) is worth nothing since the debtor has not repaid.

Under no default, the price of the bond is equal to the coupon paid today (δ) and the price of the bond issued in the future, less the payment of that coupon. In other words, the present value of the cash flows associated with the bond, both paid today and paid in the future.

Further, from equation (3) above, the price of the risk-free asset, or the reserves collapses to:

$$q_a = e^{-r}$$

From $m_{t,t+1} = e^{-r-\left(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma_\varepsilon^2\right)}$, the risk premium shock, κ_t is equal to zero, so $m_{t,t+1} = e^{-r-(0)} = e^{-r}$.

Thus, from equation (4), if $\delta = 1$ as is the case of a one period, risk-free bond/asset:

$$q_a = \mathrm{E}_{s'|s}\left[e^{-r}[1-0][1+(1-1)q'_a]\right]$$

$$q_a = e^{-r}$$

The government's problem is solved using value function iteration and the authors compute the limit of the finite horizon version of the economy.

The consumer's utility function is given by:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$$

While the utility loss function is given by:

$$U^D(y) = a_0 + a_1 \log(y)$$

The sovereign spread is defined as the difference between the yield on the bonds and the risk-free rate:

$$r_t^s = i_b - r$$

and the yield satisfies the following expression which defines the return on the bond assuming the bond is held to maturity and the government does not default:

$$q_t = \sum_{j=1}^{\infty} \delta(1-\delta)^{j-1} e^{-j i_b}$$

where i_b = yield on debt if held to maturity

Further, the country's debt levels are determined as the present value of future payments discounted at the risk-free rate and are given by:

$$\frac{\delta}{1-(1-\delta)e^{-r}}b_t$$

Bianchi et. al (forthcoming) illustrate how borrowing to accumulate reserves benefits the economy. Assume that the debt and reserve combination which yields consumption equal to its target $\bar{c}$ satisfies the equation:

$$q_a a' = y - \bar{c} - g - \delta b + a + q(a', b', s)[b' - (1-\delta)b] \qquad (5)$$

Equation (5) mirrors the government's budget constraint during non-default periods (equation 2). Further define $\tilde{a}(b', x)$ as the amount of reserves that can be purchased when the government borrows b', for a given *x* and the level of reserves (a') consistent with equation 5.

From equation (5), replace a' with $\tilde{a}(b', x)$ and apply the implicit function theorem:

$$q_a\tilde{a}(b', x) \ = y - \bar{c} - g - \delta b + a + q(\tilde{a}(b', x)\ , b', s)[b' - (1-\delta)b]$$

$$0 = y - \bar{c} - g - \delta b + a + q(\tilde{a}(b', x), b', s)[b' - (1-\delta)b] - q_a\tilde{a}(b', x)$$

The implicit function theorem suggests that:

$$\frac{\partial\tilde{a}(b', x)}{\partial b'} = -\frac{\frac{\partial F}{\partial b'}}{\frac{\partial F}{\partial\tilde{a}(b', x)}}$$

$$\frac{\partial F}{\partial b'} = \frac{\partial q(\tilde{a}(b', x), b', s)}{\partial b'}[b' - (1-\delta)b] + q(\tilde{a}(b', x), b', s)$$

$$\frac{\partial F}{\partial\tilde{a}(b', x)} = -q_a + \frac{\partial q(\tilde{a}(b', x), b', s)}{\partial a'}[b' - (1-\delta)b]$$

Therefore, combining the two yields the following two equations, the latter of which is equation (6):

$$\frac{\partial\tilde{a}(b', x)}{\partial b'} = -\frac{\frac{\partial q(\tilde{a}(b', x), b', s)}{\partial b'}[b' - (1-\delta)b] + q(\tilde{a}(b', x), b', s)}{-q_a + \frac{\partial q(\tilde{a}(b', x), b', s)}{\partial a'}[b' - (1-\delta)b]}$$

$$\frac{\partial\tilde{a}(b', x)}{\partial b'} = \frac{\frac{\partial q(\tilde{a}(b', x), b', s)}{\partial b'}[b' - (1-\delta)b] + q(\tilde{a}(b', x), b', s)}{q_a - \frac{\partial q(\tilde{a}(b', x), b', s)}{\partial a'}[b' - (1-\delta)b]} \qquad (6)$$

Finally, equation (12) of Bianchi et. al (forthcoming) defines the net benefits from increasing gross positions (both debt and reserves) as:

$$\frac{d\mathrm{E}_{s'|s}V(\tilde{a}, b', s')}{db'} \leq \frac{i}{q_a}\frac{d\tilde{q}}{db'}$$

where, finding the total derivative to the bond price with respect to bond holdings ($\frac{d\tilde{q}}{db'}$) and leveraging equation (6) where the value of a bond equals $q(\tilde{a}(b', x)\,, b', s)$ yields:

$$\frac{d\tilde{q}}{db'} = \frac{\partial q(\tilde{a}, b', s)}{\partial a'}\frac{\partial \tilde{a}}{\partial b'} + \frac{\partial q(\tilde{a}, b', s)}{\partial b'}\frac{\partial b'}{\partial b'}$$

$$\frac{d\tilde{q}}{db'} = \frac{\partial q(\tilde{a}, b', s)}{\partial a'}\frac{\frac{\partial q(\tilde{a}, b', s)}{\partial b'} i + q(\tilde{a}, b', s)}{q_a - \frac{\partial q(\tilde{a}, b', s)}{\partial a'} i} + \frac{\partial q(\tilde{a}, b', s)}{\partial b'}$$

$$\frac{d\tilde{q}}{db'} = \frac{\left[\frac{\partial q(\tilde{a}, b', s)}{\partial b'} i + q(\tilde{a}, b', s)\right]\frac{\partial q(\tilde{a}, b', s)}{\partial a'} + \frac{\partial q(\tilde{a}, b', s)}{\partial b'}\left[q_a - \frac{\partial q(\tilde{a}, b', s)}{\partial a'} i\right]}{q_a - \frac{\partial q(\tilde{a}, b', s)}{\partial a'} i}$$

$$\frac{d\tilde{q}}{db'} = \frac{q(\tilde{a}, b', s)\frac{\partial q(\tilde{a}, b', s)}{\partial a'} - \frac{\partial q(\tilde{a}, b', s)}{\partial b'} q_a}{q_a - \frac{\partial q(\tilde{a}, b', s)}{\partial a'} i}$$

Thus, ultimately, Bianchi et. al's (forthcoming) analysis leads us to the conclusion that the value of $\frac{i}{q_a}\frac{d\tilde{q}}{db'}$ and thus the net benefits of issuing debt to accumulate reserves depends on whether the effects on the value of debt (and correspondingly on yields and spreads) of increasing reserves outpaces the effects of greater debt holdings on the value of existing debt. Thus, there likely comes a point, above which, borrowing to hold reserves provides no additional benefits to the country.

APPENDIX A3: THEORETICAL FRAMEWORK FROM LEVY YEYATI (2008)

Levy Yeyati (2008) shed some light on a potential specification and highlight that the marginal cost of issuing debt depends on the responsiveness of the risk premium to reserves and debt respectively as well as to the respective levels of reserves and debt. According to them, the risk neutral investor holds a bond such that:

$$\left(1+r_f\right)=\left(1+r_f+\rho\right)[1-p(R,D)]+\left(1+r_f+\rho\right)(1-H)\times p(R,D)$$

where r_f represents the risk-free interest rate, ρ is the risk premium, H is the haircut applied to the debt in percentage terms in default and $p(R,D)$ captures the probability of default which depends on both reserves (R) and debt (D). Rearranging gives:

$$\frac{H\times p(R,D)}{[1-H\times p(R,D)]}\left(1+r_f\right)=\rho$$

Finding the first derivatives of the risk premium with respect to reserves and debt yields:

$$\frac{\partial\rho}{\partial R}=\rho_R(R,D)=\frac{H\times p_R(R,D)}{[1-H\times p(R,D)]^2}\left(1+r_f\right)$$

$$\frac{\partial\rho}{\partial D}=\rho_D(R,D)=\frac{H\times p_D(R,D)}{[1-H\times p(R,D)]^2}\left(1+r_f\right)$$

Simplifying and setting equal to each other, we have:

$$\rho_R(R,D)\div\frac{H\times p_R(R,D)}{[1-H\times p(R,D)]^2}=\left(1+r_f\right)$$

$$\rho_D(R,D)\div\frac{H\times p_D(R,D)}{[1-H\times p(R,D)]^2}=\left(1+r_f\right)$$

$$\rho_R(R,D)\div\frac{H\times p_R(R,D)}{[1-H\times p(R,D)]^2}=\rho_D(R,D)\div\frac{H\times p_D(R,D)}{[1-H\times p(R,D)]^2}$$

$$\rho_R(R,D)\times\frac{[1-H\times p(R,D)]^2}{H\times p_R(R,D)}=\rho_D(R,D)\times\frac{[1-H\times p(R,D)]^2}{H\times p_D(R,D)}$$

$$\rho_R(R,D)=\rho_D(R,D)\times\frac{[1-H\times p(R,D)]^2}{H\times p_D(R,D)}\div\frac{[1-H\times p(R,D)]^2}{H\times p_R(R,D)}$$

$$\rho_R(R,D)=\rho_D(R,D)\times\frac{[1-H\times p(R,D)]^2}{H\times p_D(R,D)}\times\frac{H\times p_R(R,D)}{[1-H\times p(R,D)]^2}$$

$$\rho_R(R,D) = \rho_D(R,D) \times \frac{H \times p_R(R,D)}{H \times p_D(R,D)} = \rho_D(R,D) \times \frac{p_R(R,D)}{p_D(R,D)} \leq 0$$

Given that we assume that reserves reduce the probability of default, then the above expression will be less than or equal to 0.

Additionally, the last expression above suggests that the marginal change in the risk premium from a unit change in reserves may equal the absolute value of the marginal change in the risk premium from a unit change in debt if the marginal changes in the probabilities are equal (and the ratio of them is equal to 1). However, if an additional unit of reserves reduces the probability of default more than a unit of debt increases the probability of default, then the marginal effect of reserves on spreads is greater than the marginal effect of debt on spreads.

The government's loss function can be expressed as:

$$L(R,D) = \left[r_f + \rho(R,D)\right]D + p(R,D)K - r_f R + k$$

where *K* captures the expected cost of a crisis and *k* measures other factors independent of reserves and debt stocks.

Assuming each additional unit of reserves is financed via an additional unit of debt then:

$$\frac{\partial D}{\partial R} = 1$$

Therefore, the first derivative of the government's loss function of a change in reserves financed via a change in debt gives:

$$\frac{\partial L(R,D)}{\partial R} = L_R(R,D) + L_D(R,D)$$

$$L_R(R,D) = \rho_R(R,D)D + p_R(R,D)K - r_f$$

$$L_D(R,D) = r_f + \rho(R,D) + \rho_D(R,D)D + p_D(R,D)K$$

$$\frac{\partial L(R,D)}{\partial R} = L_R(R,D) + L_D(R,D)$$
$$= \rho_R(R,D)D + p_R(R,D)K + \rho(R,D) + \rho_D(R,D)D + p_D(R,D)K$$

$$\frac{\partial L(R,D)}{\partial R} = p_R(R,D)K + p_D(R,D)K + \rho(R,D) + \rho_R(R,D)D + \rho_D(R,D)D$$

$$\frac{\partial L(R,D)}{\partial R} = [p_R(R,D) + p_D(R,D)]K + \rho(R,D) + [\rho_R(R,D) + \rho_D(R,D)]D$$

This last expression gives the net marginal benefit of more reserves (likely a net reduction in the probability of a default) plus the marginal cost of borrowing which is the spread on the debt less the marginal reduction in the spread on existing debt. The net marginal cost of borrowing to buy a unit of reserves is therefore:

$$C'(R,D) = \rho(R,D) + [\rho_R(R,D) + \rho_D(R,D)]D$$

If we define the percentage changes in spreads with respect to changes in reserves and debt as:

$$\frac{\rho_R(R,D)}{\rho(R,D)} \times R = \beta_R$$

$$\frac{\rho_D(R,D)}{\rho(R,D)} \times D = \beta_D$$

Thus, the sensitivities of spreads to reserves and debt each depend on the existing levels of reserves and debt respectively. So, if debt levels are already high, the responsiveness of spreads to changes in debt are higher. Similarly, the responsiveness of spreads to reserves varies with the existing reserve level.

Thus:

$$C'(R,D) = \rho(R,D) + \left[\frac{\beta_R \times \rho(R,D)}{R} + \frac{\beta_D \times \rho(R,D)}{D}\right] D$$

$$C'(R,D) = \rho(R,D) + \left[\frac{\beta_R \times \rho(R,D)}{R} D + \frac{\beta_D \times \rho(R,D)}{D} D\right]$$

$$C'(R,D) = \rho(R,D) + \left[\frac{\beta_R \times \rho(R,D)}{R} D + \beta_D \times \rho(R,D)\right]$$

$$C'(R,D) = \rho(R,D) + \rho(R,D)\left[\frac{\beta_R}{R} D + \beta_D\right]$$

Let $\omega = \frac{R}{D}$ then:

$$C'(R,D) = \rho(R,D)\left[1 + \left(\frac{\beta_R}{\omega} + \beta_D\right)\right]$$

Hence, the marginal cost of issuing debt to purchase reserves is a function of the current spread on debt, the responsiveness or spreads to reserves and debt respectively and the ratio of reserves to debt. It would make sense to issue debt until the marginal cost of doing so is equal to the marginal revenue of issuing debt to purchase reserves.

Holding the marginal revenue constant, a country would not want the marginal cost to be positive. Thus, countries would wish to issue debt to purchase reserves up until the marginal cost is equal to 0. From the marginal cost equation above, this requires: $1 + \left(\frac{\beta_R}{\omega} + \beta_D\right) = 0$ or $\left(\frac{\beta_R}{\omega} + \beta_D\right) = -1$

APPENDIX A4: PARTICIPATING COUNTRIES AND AVERAGE POPULATION

Country Name	Average Population Size
Argentina	42,105,104
Belize	336,635
Brazil	200,461,745
Colombia	46,837,883
Dominican Republic	10,149,848
Ecuador	15,418,294
El Salvador	6,223,908
Gabon	1,757,804
Georgia	3,841,489
Ghana	25,740,865
Indonesia	248,783,952
Jamaica	2,838,787
Kazakhstan	16,788,742
Lebanon	4,985,960
Mexico	120,732,583
Pakistan	178,132,556
Panama	3,773,935
Peru	30,180,630
Philippines	96,940,423
Russia	143,367,281
Serbia	7,204,895
South Africa	53,093,667
Sri Lanka	20,498,254
Turkey	74,743,035
Ukraine	45,600,202
Venezuela	29,876,996
Vietnam	90,535,280