Principal Component Value at Risk: an application to the measurement of the interest rate risk exposure of Jamaican Banks to Government of Jamaica (GOJ) Bonds

Mark Tracey
Financial Stability Department
Research & Economic Programming Division
Bank of Jamaica

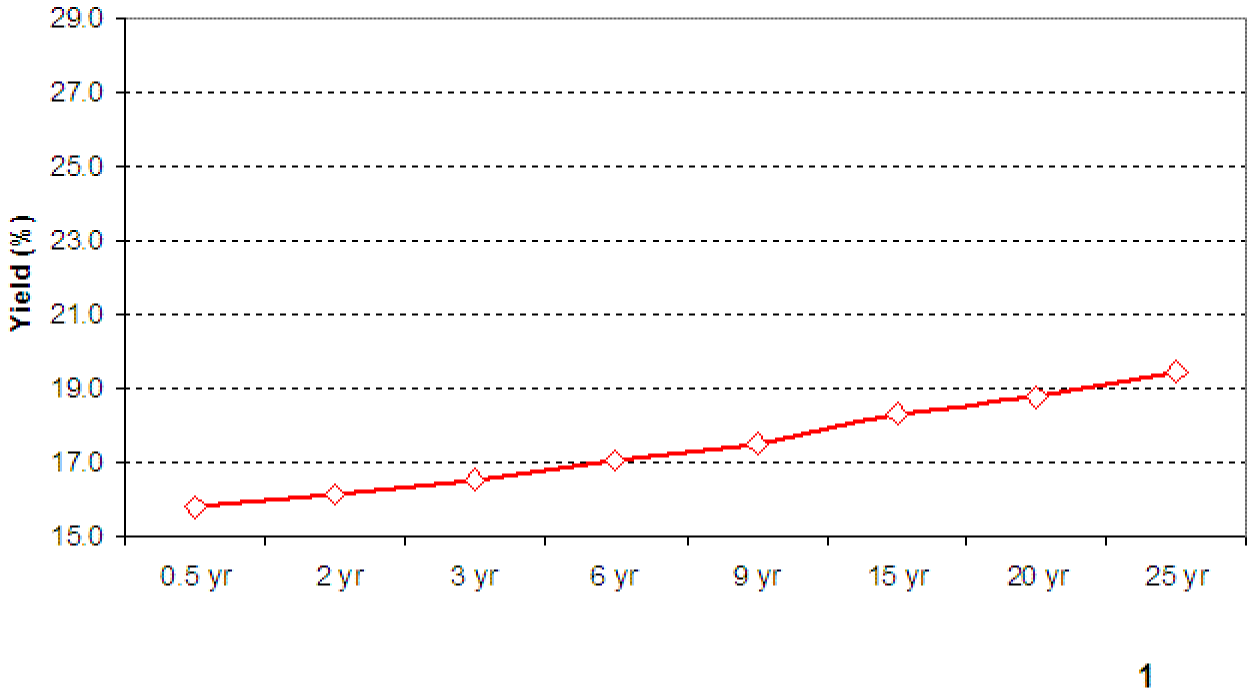
Presentation Format

- Introduction
- Motivation
- Data
- Methodology
- Results
- Conclusion
- Policy Implication
- Future Work

Introduction

Movements in Domestic Yield Term Structure

4th August 2008 - 31st December 2008



Introduction

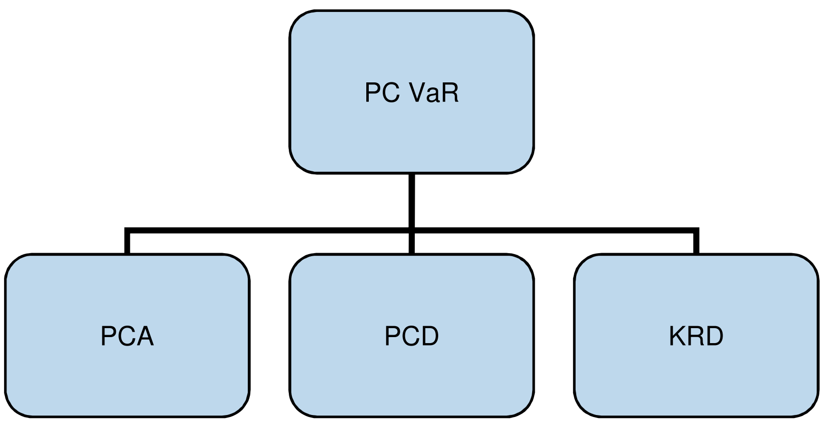
- In finance a portfolio's risk is typically measured by Value at Risk (VaR)
  - *"VaR measures the worst expected loss over a given horizon under normal market conditions at a given confidence level"* (Jorion, 2004).
- In using VaR methodology risk managers may face a number of challenges:
  - As asset number increase some correlations will be measured inaccurately or incorrectly.
  - Computation time of covariance matrix and subsequent VaR calculations can increase dramatically.
  - Issues of normality.
- As an alternative Principal Component Analysis (PCA) often used in portfolio risk management.
- What is PCA ?

Introduction

- PCA, is a widely used technique in portfolio risk management which reduces the amount of risk factors driving a portfolio.

- The study therefore computes a VaR outturn that incorporates PCA.

- In arriving at a PC VaR the paper combines:



Motivation

- Incorporating a framework that can:
  - Measure and monitor interest rate risk for data that is not normally distributed.
  - Reduce the size of the covariance matrix.
  - Provide information regarding the impact of each risk factor.

Previous Studies

- Principal component analysis was first applied to fixed income market by Garbade (1997).
- Jatnshidian and Zhu (1997) applied PCA to fixed-income portfolios.
- Loretan (1997) and Frye (1997) apply PCA in the context of VAR methodology.
- Wu (2003) used PCA to show that three factors are main drivers behind term structure movements denoted as level, slope and curvature factors.
- Malava (2006) through the use of scenario based PCA to reduced the dimensionality of currency movement across currency zones.

Data

- Government of Jamaica (GOJ) global bond yields from 23 February 2006 to 18 March 2009 for securities:
  - 7-year
  - 9-year
  - 20-year
  - 30-year

- GOJ domestic bonds yields from 3 January 2008 to 18 March 2009 for securities:
  - 6-month
  - 2-year
  - 3-year
  - 6-year
  - 9-year
  - 15-year
  - 20-year
  - 25-year

- Holdings of GOJ securities by each banking institution was obtained from re-pricing data for the banking system as at end December 2008.

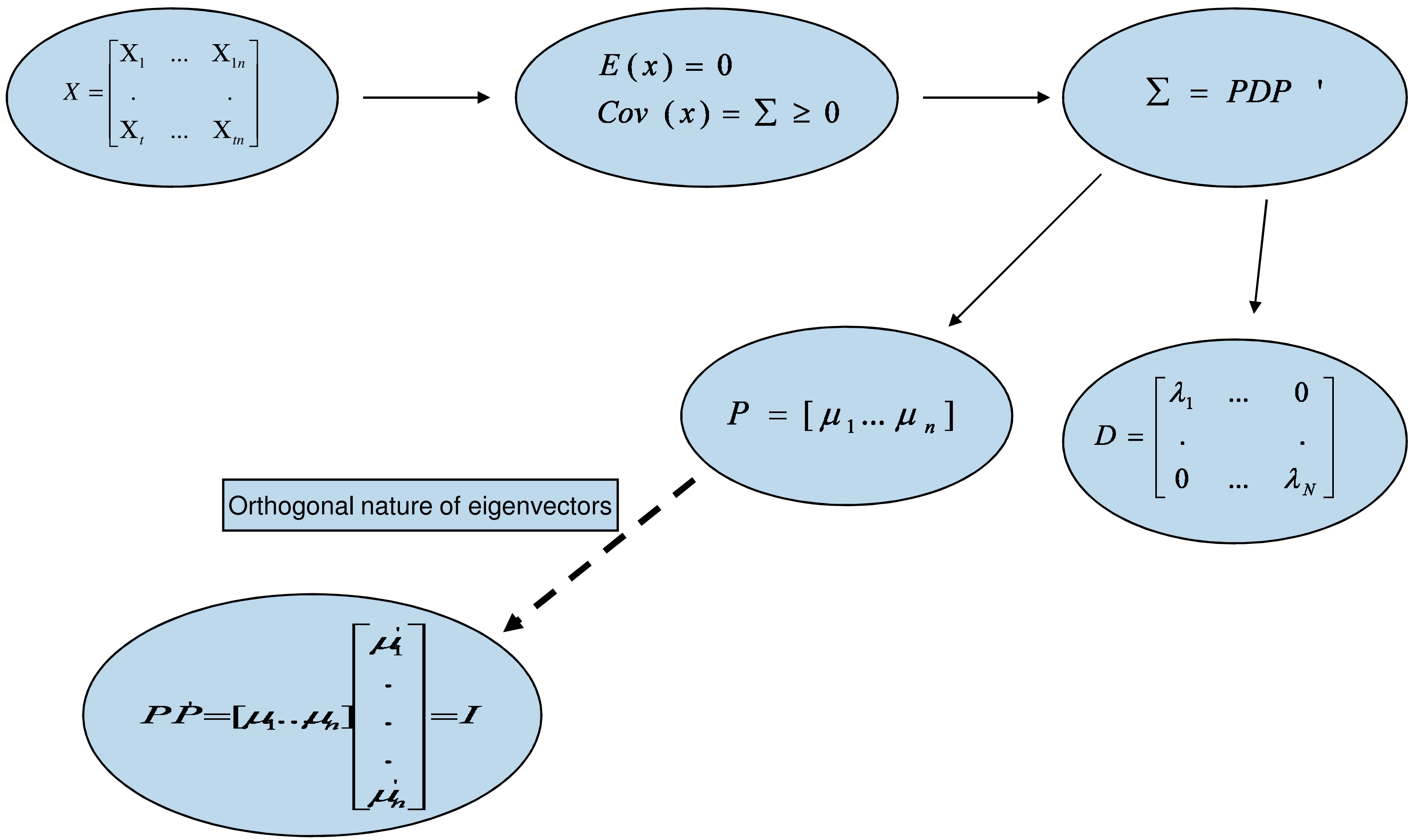
Methodology

Statistical Analysis of data

- Simple statistical tests.
- Augmented-Dickey Fuller test.
- Jarque-Bera test.
- Correlation Matrix.

Methodology cont'd

Principal Components



Methodology cont'd

Principal Components cont'd

- Singular value decomposition equation, which decomposes original matrix as:

$$\Sigma = PDP' = [e_1 \ldots e_n]\begin{bmatrix}\lambda_1 & \ldots & 0\\ \vdots & & \vdots \\ 0 & \ldots & \lambda_n\end{bmatrix}\begin{bmatrix} e_1' \\ \vdots \\ e_n'\end{bmatrix} = \lambda_1 e_1 e_1' + \ldots + \lambda_n e_n e_n'$$

where P is an orthogonal matrix, i.e., such that its inverse is also its transpose, $PP' = I$ and D a diagonal matrix composed of the $\lambda_1's$.

Principal Components cont'd

- The following criterion is usually used to ascertain the number of PCs to be adopted:

$$\frac{\lambda_1 + \ldots + \lambda_k}{\lambda_1 + \ldots + \lambda_n} > 1 - \varepsilon^*$$

Where $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n$ are the eigenvalues of pc and 1-e is the threshold level.

Methodology cont'd

Principal Components cont'd

- Interest rates can be now be expressed by equation:

$$\Delta y(t_i) = l_{ih}\Delta c_h + l_{is}\Delta c_s + l_{ic}\Delta c_c$$

- The principal components are defined as follows:

$$\Delta c_h = \Delta c_1^* = \frac{\Delta c_1}{\sqrt{\lambda_1}}, \qquad \Delta c_s = \Delta c_2^* = \frac{\Delta c_2}{\sqrt{\lambda_2}}, \qquad \Delta c_c = \Delta c_3^* = \frac{\Delta c_3}{\sqrt{\lambda_3}},$$

- The factor loadings are defined as follows:

$$l_{ih} = u_1\sqrt{\lambda_1}, \qquad l_{is} = u_{2i}\sqrt{\lambda_2}, \qquad l_{ic} = u_{3i}\sqrt{\lambda_3},$$

Where $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n$ are the eigenvalues of c, ranked in decreasing order, and $u_1, \mu_2, \ldots \mu_n$ is the corresponding eigenvectors.

Methodology cont'd

Key Rate Duration

- The KRD of each bond computed by equation:

$$KRD(i) = \frac{1}{p}\frac{CF_i \times t_i}{e^{t_i \times y(t_i)}}$$

Where KRD (i) is the i^{th} key rate duration, *p* is is the price of the bond, is the i^{th} cash flow, is the i^{th} time period.

Methodology cont'd

Principal Components Duration

- Once the principal components have been indentified, the PCDs are computed using the equation:

$$PCD(v) = \sum_{i=1}^{m} KRD(i) \times l_{iv}$$

Note that v Indicates whether the height, slope or curvature is being calculated.

Methodology cont'd

Principal Components Duration cont'd

- The portfolio can be immunized using the PC model:

$$PCD(h) = p_1 \times PCD_1(h) + p_2 \times PCD_2(h) + ...p_n \times PCD_n(h) = H \times l_{Hh}$$

$$PCD(s) = p_1 \times PCD_1(s) + p_2 \times PCD_2(s) + ...p_n \times PCD_n(s) = H \times l_{Hs}$$

$$PCD(c) = p_1 \times PCD_1(c) + p_2 \times PCD_2(c) + ...p_n \times PCD_n(c) = H \times l_{Hc}$$

$$p_1 + p_2 + ... p_n = 1$$

Where p represents the proportion of various types of bonds held in the portfolio.

Methodology cont'd

VaR using Principal Component Duration

- The 99th per cent VaR for each portfolio using principal component model was calculated using equation:

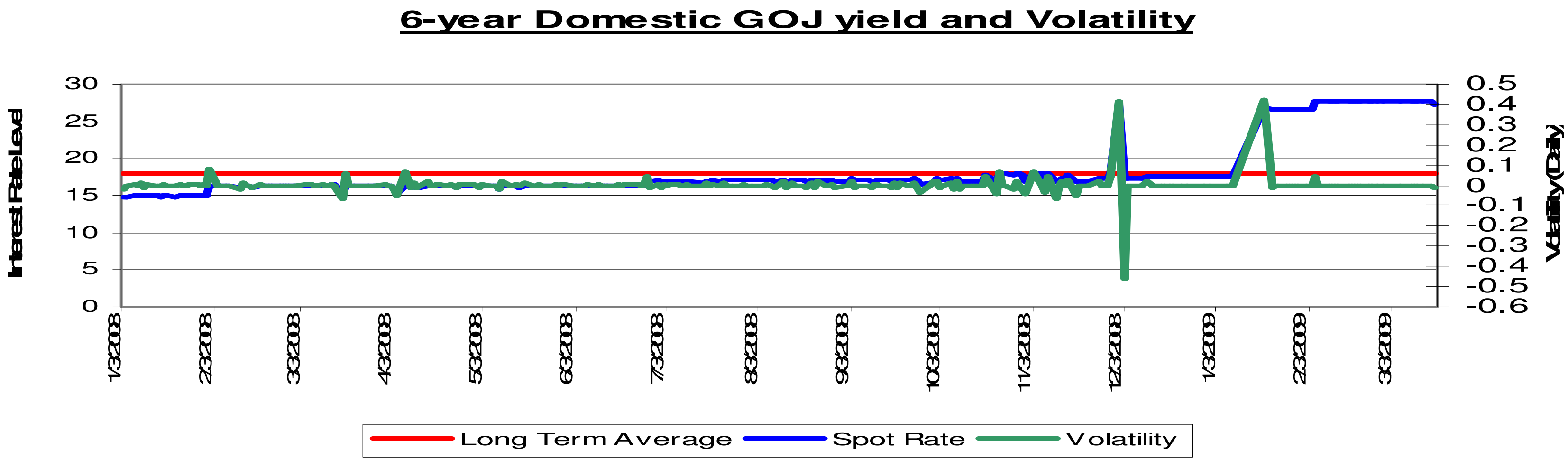
$$10-dayPCVaR_{99} = \left(\Gamma \times 2.326 \times \sqrt{PCD_{port}(h)^2 + PCD_{port}(s)^2 + PCD_{port}(c)^2}\right)\sqrt{10}$$

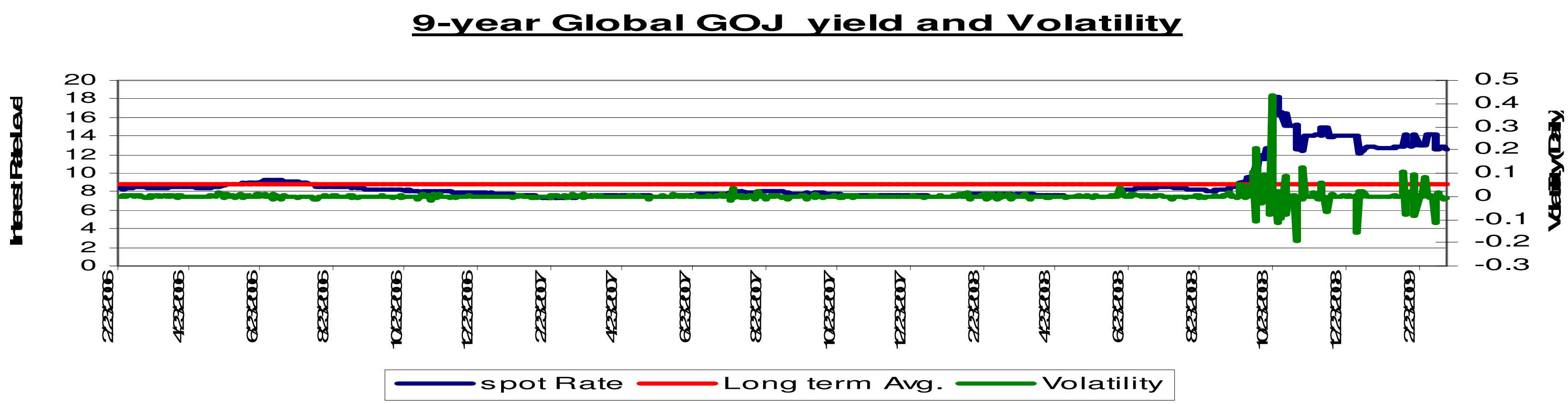
Where Γ is the market value of the portfolio and the 99th percentile of a standard distribution is 2.326.

- A 10-day parametric and non-parametric VaR were also computed.

Results

Statistical Analysis





Results cont'd

Statistical Analysis **cont'd**

GOJ Domestic par Yield Curve-Descriptive Statistics (April 1 2008 to March 18 2009)

	0.5 yr	2 yr	3 yr	6 yr	9 yr	15 yr	20 yr	25 yr
Mean	0.0019	0.0023	0.0024	0.0024	0.0023	0.0007	0.0006	0.0023
Median	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	0.0523	0.3561	0.3708	0.4226	0.4215	0.4258	0.4335	0.4000
Minimum	-0.0314	-0.2188	-0.1089	-0.4571	-0.4429	-0.4294	-0.4274	-0.2079
Std. Dev.	0.0091	0.0346	0.0282	0.0498	0.0489	0.0399	0.0417	0.0449
Skewness	1.5072	4.1194	9.0189	1.5190	1.8814	-0.1383	0.2058	3.5308
Kurtosis	10.6597	57.1076	118.8034	67.7900	69.1866	106.1529	91.5266	36.8863
Jarque-Bera	711.4539	31452.8500	144225.7000	44173.1800	46145.6000	111726.3000	82289.8900	12580.5500
Augmented Dickey-Fuller	-8.228609	-16.61035	-17.91529	-14.92002	-21.80277	-14.87754	-15.8462	-22.85541
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Sum	0.4762	0.5771	0.6021	0.6070	0.5726	0.1800	0.1531	0.5695
Sum Sq. Dev.	0.0208	0.3011	0.1997	0.6232	0.5995	0.3987	0.4355	0.5066
Observations	252.0000	252.0000	252.0000	252.0000	252.0000	252.0000	252.0000	252.0000

***Dickey-Fuller unit root test: 5% critical Value is equal to -2.87**

Results cont'd

Statistical Analysis **cont'd**

GOJ Global par Yield Curve-Descriptive Statistics (February 24 2006 to March 18 2009)

	7 yr	9 yr	20 yr	30 yr
Mean	0.000537	0.000527	0.0006	0.000595
Median	0	0	0	0
Maximum	0.159839	0.435113	0.211864	0.329919
Minimum	-0.190533	-0.192492	-0.202918	-0.324603
Std. Dev.	0.017502	0.024475	0.019449	0.029207
Skewness	0.569684	6.860503	0.912079	1.162565
Kurtosis	45.39222	144.4259	75.55154	80.85242
Jarque-Bera	59571.95	668779	174471.1	200949.7
Augmented Dickey-Fuller	-4.544742	-5.844139	-21.60681	-4.304693
Probability	0	0	0	0
Sum	0.427145	0.419288	0.476863	0.472645
Sum Sq. Dev.	0.24321	0.475622	0.300342	0.677316
Observations	795	795	795	795

***Dickey-Fuller unit root test: 5% critical Value is equal to-2.86**

Results cont'd

Statistical Analysis **cont'd**

Correlation Matrix for Global Bonds

	7 yr	9 yr	20 yr	30 yr
7 yr	1.00			
9 yr	0.94	1.00		
20 yr	0.98	0.96	1.00	
30 yr	0.96	0.91	0.97	1.00

Correlation matrix of Domestic bonds

	0.5 yr	2 yr	3 yr	6 yr	9 yr	15 yr	20 yr	25 yr
0.5 yr	1.00							
2 yr	0.92	1.00						
3 yr	0.93	0.97	1.00					
6 yr	0.96	0.92	0.90	1.00				
9 yr	0.95	0.92	0.89	1.00	1.00			
15 yr	0.51	0.58	0.51	0.55	0.58	1.00		
20 yr	0.43	0.48	0.42	0.47	0.50	0.93	1.00	
25 yr	0.87	0.95	0.96	0.84	0.84	0.50	0.42	1.00

Results cont'd

Examination of Re-pricing Schedule

The Re-pricing gap Domestic Assets Structure (end-December 2008).

	91 - 365 days	1 - 2 yrs	2 - 5 yrs	5 - 10 yrs	10 - 15 yrs	15 - 20 yrs	over 20 yrs
Commercial Banks							
CBank 1	12.90%	37.56%	36.60%	6.84%	4.38%	0.00%	1.72%
CBank 2	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CBank 3	0.00%	7.56%	82.48%	9.96%	0.00%	0.00%	0.00%
CBank 4	64.98%	0.00%	35.02%	0.00%	0.00%	0.00%	0.00%
CBank 5	84.89%	0.12%	10.58%	2.23%	0.32%	1.86%	0.00%
CBank 6	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CBank 7	38.82%	0.00%	51.20%	8.15%	1.82%	0.00%	0.00%
Merchant Banks							
Mbank 1	16.20%	34.70%	0.55%	7.63%	40.91%	0.00%	0.00%
MBank2	0.00%	32.65%	67.35%	0.00%	0.00%	0.00%	0.00%
MBank3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Building Societies							
BSoc1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
BSoc2	86.76%	11.70%	0.00%	1.53%	0.00%	0.00%	0.00%
BSoc3	98.02%	0.00%	1.98%	0.00%	0.00%	0.00%	0.00%
BSoc4	95.22%	0.00%	4.78%	0.00%	0.00%	0.00%	0.00%

Results cont'd

Examination of Re-pricing Schedule

The Re-pricing gap FX Assets Structure (end-December 2008).							
	91 - 365 days	1 - 2 yrs	2 - 5 yrs	5 - 10 yrs	10 - 15 yrs	15 - 20 yrs	over 20 yrs
Commercial Banks							
CBank 1	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 2	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 3	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 4	0.000%	0.000%	73.886%	26.114%	0.000%	0.000%	0.000%
CBank 5	4.329%	4.136%	42.691%	29.459%	19.385%	0.000%	0.000%
CBank 6	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
CBank 7	5.887%	37.791%	0.000%	16.002%	40.320%	0.000%	0.000%
Merchant Banks							
Mbank 1	0.000%	8.939%	1.039%	12.643%	12.623%	20.090%	44.665%
MBank2	0.000%	79.730%	20.270%	0.000%	0.000%	0.000%	0.000%
MBank3	24.176%	0.000%	57.497%	8.910%	9.416%	0.000%	0.000%
Building Societies							
BSoc1	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
BSoc2	11.480%	74.568%	0.000%	13.952%	0.000%	0.000%	0.000%
BSoc3	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
BSoc4	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%

Results cont'd

Sensitivity Factors of Instruments

SENSITIVITY FACTORS OF INSTRUMENTS					
	Domestic		Global		
Maturity	**Duration**	**Convexity**	**Maturity**	**Duration**	**Convexity**
6 mm	0.5	0.250	**7 yr**	4.889	28.994
2 yr	1.8	3.401	**9 yr**	5.466	38.782
3 yr	2.433	6.561	**20 yr**	8.116	101.420
6 yr	3.908	19.006	**30 yr**	12.472	214.455
9 yr	4.884	32.352			
15 yr	5.714	50.612			
20 yr	6.164	62.733			
25 yr	7.093	82.544			

Results cont’d

Principal Component Analysis cont’d

Domestic Factor Loading

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
0.5 yr	-0.0001	0.000	0.000	0.000	-0.002	-0.001	0.001	0.009
2 yr	0.025	-0.001	-0.004	0.007	0.020	-0.002	0.003	0.000
3 yr	0.003	-0.002	-0.010	0.028	-0.006	0.000	0.000	0.000
6 yr	0.043	-0.002	-0.020	-0.005	0.000	0.005	-0.007	0.001
9 yr	0.042	-0.001	-0.016	-0.006	-0.008	-0.004	0.007	-0.001
15 yr	0.041	0.002	0.018	0.002	-0.002	-0.008	-0.005	0.000
20 yr	0.042	0.002	0.021	0.003	-0.002	0.008	0.003	0.000
25 yr	0.000	0.043	-0.004	0.001	0.000	0.000	0.000	0.000

	PC1	PC2	PC3
0.5 yr	-0.0001	0.000	0.000
2 yr	0.025	-0.001	-0.004
3 yr	0.003	-0.002	-0.010
6 yr	0.043	-0.002	-0.020
9 yr	0.042	-0.001	-0.016
15 yr	0.041	0.002	0.018
20 yr	0.042	0.002	0.021
25 yr	0.000	0.043	-0.004

Results cont'd

Principal Component Analysis cont'd

PORTION OF VARIANCE EXPLAINED BY FIRST THREE PC						
	Domestic			Global		
	PC1	PC2	PC3	PC1	PC2	PC3
Eigenvalue	0.008	0.002	0.002	0.560	0.112	0.055
Variability (%)	59.761	14.484	12.002	73.482	14.646	7.212
Cumulative (%)	59.761	74.244	86.246	73.482	88.128	95.339

Results cont'd

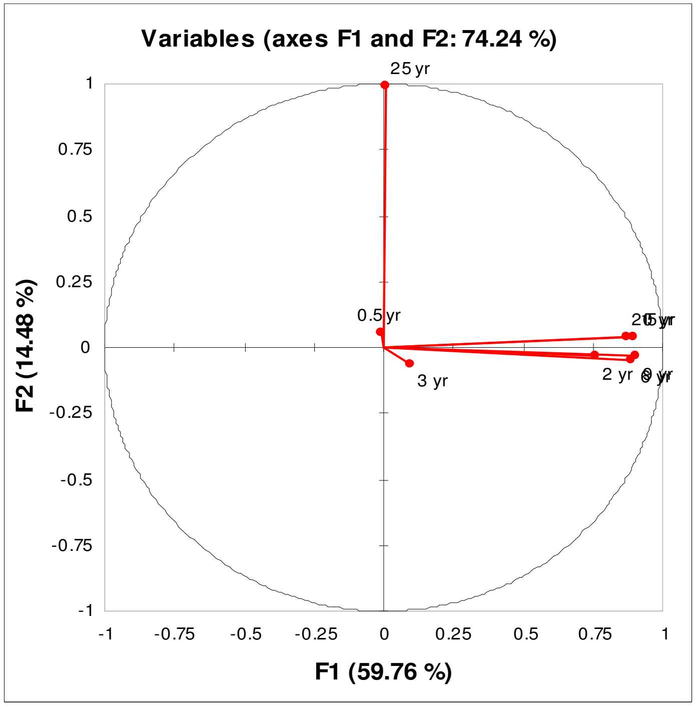
Principal Component Analysis cont'd

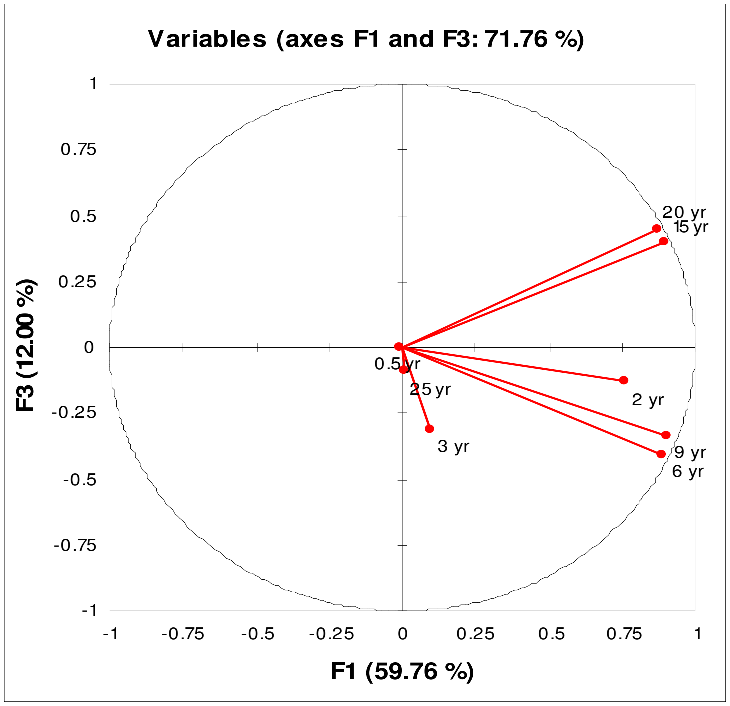
PCA ON DAILY BASIS: FACTOR LOADINGS							
	Domestic			Global			
Maturity	**PC1**	**PC2**	**PC3**	**Maturity**	**PC1**	**PC2**	**PC3**
6 m	-0.00005	0.00047	-0.00003				
2 yr	0.025	-0.001	-0.004				
3 yr	0.003	-0.002	-0.010				
6 yr	0.043	-0.002	-0.020	**7 yr**	-0.034	0.007	0.056
9 yr	0.042	-0.001	-0.016	**9 yr**	0.000	0.333	-0.022
15 yr	0.041	0.002	0.018				
20 yr	0.042	0.002	0.021	**20 yr**	0.002	0.031	0.226
25 yr	0.000	0.043	-0.004	**30 yr**	0.748	0.000	0.002

Results cont’d

Principal Component Analysis cont’d

Diagrammatic representation of correlations between variables and factors for Domestic Bonds

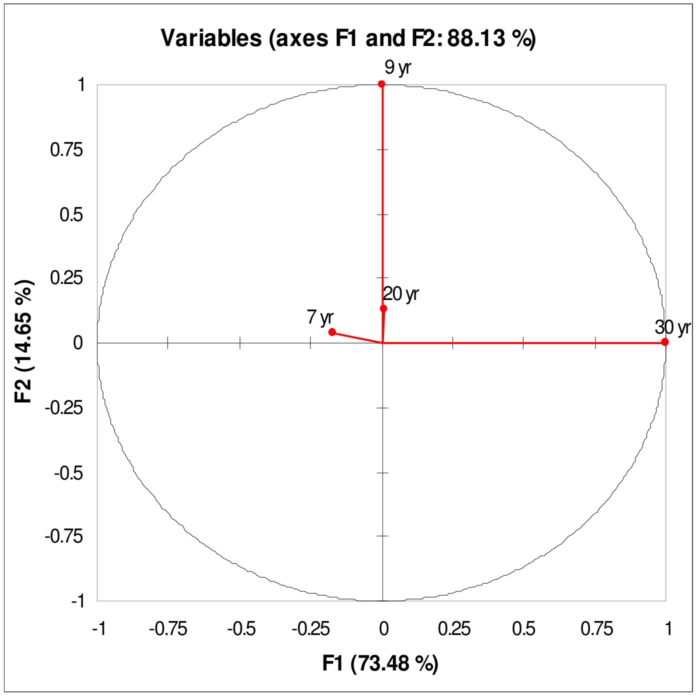


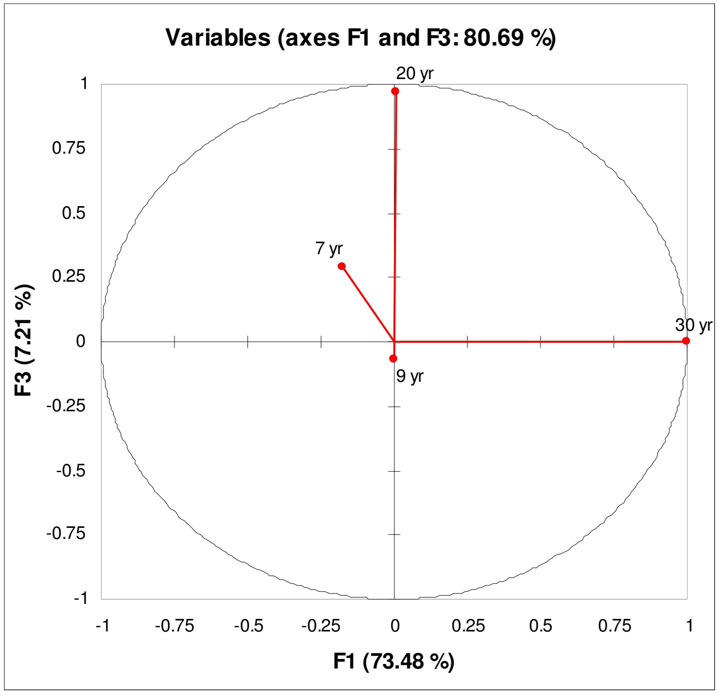


Results cont'd

Principal Component Analysis cont'd

Diagrammatic representation of correlations between variables and factors for Global Bonds





Results cont'd

Value at Risk Outturn

Comparison of Different Risk Measures for Domestic GOJ Bonds: 10-day PC VaR versus 10-day VaR

	10-day Principal Component VaR	10-day VaR	
		Parametric VaR	Non Parametric VaR
Commercial Banks			
CBank 1	-26.2%	-10.2%	-15.5%
CBank 2	0.0%	0.0%	0.0%
CBank 3	-31.4%	-14.4%	-25.7%
CBank 4	-2.6%	-2.0%	-2.4%
CBank 5	-4.8%	-2.6%	-5.3%
CBank 6	-0.1%	-0.3%	-0.4%
CBank 7	-7.7%	-3.6%	-7.0%
Total	-6.8%	-2.1%	-3.3%
Merchant Banks			
Mbank 1	-39.6%	-26.9%	-17.4%
Mbank 2	-20.3%	-10.3%	-13.2%
Mbank 3	0.0%	0.0%	0.0%
Total	-36.7%	-22.6%	-27.3%
Building Societies			
Bsoc 1	0.0%	0.0%	0.0%
Bsoc 2	-1.4%	-0.5%	-0.6%
Bsoc 3	-0.2%	-0.4%	-0.5%
Bsoc 4	-0.3%	-0.3%	-0.5%
Total	-0.9%	-0.4%	-0.6%
Mean	-9.6%	-5.1%	-6.3%

Results cont'd

Value at Risk Outturn cont'd

Comparison of Different risk Measures on Global GOJ Bonds:10-Day PC VaR versus 10-day VaR

	10-day Principal Component VaR	10-day VaR	
		Parametric VaR	Non Parametric VaR
Commercial Banks			
CBank 1	0%	0%	0%
CBank 2	0%	0%	0%
CBank 3	0%	0%	0%
CBank 4	-7.0%	-3.9%	-5.2%
CBank 5	-7.6%	-4.2%	-5.6%
CBank 6	0%	0%	0%
CBank 7	-4.3%	-2.4%	-3.2%
Total	-7.3%	-5.5%	-6.2%
Merchant Banks			
Mbank 1	-21.7%	-26.3%	-59.7%
Mbank 2	0%	0%	0%
Mbank 3	-2.4%	-1.3%	-1.8%
Total	-20.4%	-20.7%	-23.2%
Building Societies			
Bsoc 1	0%	0%	0%
Bsoc 2	-3.8%	-2.1%	-2.8%
Bsoc 3	0%	0%	0%
Bsoc 4	0%	0%	0%
Total	-1.4%	-1.0%	-1.1%
Mean	-3.4%	-2.9%	-5.6%

Conclusion

- Method is intuitive and explains risk associated with portfolios using three factors that affect yield curves.
- PC VaR is better suited for portfolios that have a large number of assets.
- Results indicate that the risk involved with holding GOJ domestic bonds is greater than holding global GOJ bonds.

Policy Implication

- Central Banks incorporate the use of PC VaR modeling technique in monitoring risk associated with interest rate movement and its impact on banking system.

Future Work

- The use of PC VaR on entire portfolio content.

- Scenario based stress testing through the employment of Monte Carlo techniques in PC VaR framework.

The End

☹